

$$x \prod_{k=1}^{\infty} \left(1 - \frac{x^2}{k^2 \pi^2}\right) = \sin(x) \quad \text{we will find p derivative} \quad p' \left[x \prod_{k=1}^{\infty} \left(1 - \frac{x^2}{k^2 \pi^2}\right) \right] = p'(\sin(x)) \rightarrow e^{\frac{1}{x} \left(\prod_{k=1}^{\infty} e^{\frac{2x}{x^2 - k^2 \pi^2}} \right)} = e^{\frac{1}{\tan(x)}} \rightarrow \frac{1}{x} + \sum_{k=1}^{\infty} \frac{2x}{x^2 - k^2 \pi^2} = \frac{1}{\tan(x)} \quad x = \frac{\pi}{4}$$

$$+ \quad \text{we will find L derivative} \quad L' \left[x \prod_{k=1}^{\infty} \left(1 - \frac{x^2}{k^2 \pi^2}\right) \right] = L'(\sin(x)) \rightarrow 1 + \sum_{k=1}^{\infty} \frac{2x^2}{x^2 - k^2 \pi^2} = \frac{x}{\tan(x)} \rightarrow \frac{1}{x} + \sum_{k=1}^{\infty} \frac{2x}{x^2 - k^2 \pi^2} = \frac{1}{\tan(x)}$$

$$\frac{4}{\pi} + \sum_{k=1}^{70} \frac{1}{\pi(0.125 - 2k^2)} = 1.002257489 \quad \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 1$$

$$\prod_{k=1}^{\infty} \left[1 - \frac{4x^2}{(2k-1)^2 \pi^2}\right] = \cos(x) \quad \text{we will find p derivative} \quad p' \left[\prod_{k=1}^{\infty} \left[1 - \frac{4x^2}{(2k-1)^2 \pi^2}\right] \right] = p'(\cos(x)) \rightarrow \prod_{k=1}^{\infty} e^{\frac{-8x}{(2k-1)^2 \pi^2 - 4x^2}} = e^{-\tan(x)} \rightarrow \sum_{k=1}^{\infty} \frac{8x}{(2k-1)^2 \pi^2 - 4x^2} = \tan(x) \quad x = \frac{\pi}{4}$$

$$\text{we will find L derivative} \quad L' \left[\prod_{k=1}^{\infty} \left[1 - \frac{4x^2}{(2k-1)^2 \pi^2}\right] \right] = L'(\cos(x)) \rightarrow \sum_{k=1}^{\infty} \frac{-8x^2}{(2k-1)^2 \pi^2 - 4x^2} = -x \tan(x) \rightarrow \sum_{k=1}^{\infty} \frac{8x}{(2k-1)^2 \pi^2 - 4x^2} = \tan(x)$$

$$\sum_{k=1}^{70} \frac{2}{\pi \left[(2k-1)^2 - \frac{1}{4} \right]} = 0.997726387 \quad \tan\left(\frac{\pi}{4}\right) = 1$$

$$\prod_{k=1}^{\infty} \left[1 + \frac{4x^2}{(2k-1)^2 \pi^2} \right] = \cosh(x) \quad \text{we will find p derivative} \quad p' \left[\prod_{k=1}^{\infty} \left[1 + \frac{4x^2}{(2k-1)^2 \pi^2} \right] \right] = p'(\cosh(x)) \rightarrow \prod_{k=1}^{\infty} e^{\frac{8x}{(2k-1)^2 \pi^2 + 4x^2}} = e^{\tanh(x)} \rightarrow \sum_{k=1}^{\infty} \frac{8x}{(2k-1)^2 \pi^2 + 4x^2} = \tanh(x) \quad x=1$$

$$\text{we will find L derivative} \quad L' \left[\prod_{k=1}^{\infty} \left[1 + \frac{4x^2}{(2k-1)^2 \pi^2} \right] \right] = L'(\cosh(x)) \rightarrow \sum_{k=1}^{\infty} \frac{8x^2}{(2k-1)^2 \pi^2 + 4x^2} = x \tanh(x) \rightarrow \sum_{k=1}^{\infty} \frac{8x}{(2k-1)^2 \pi^2 + 4x^2} = \tanh(x)$$

$$\sum_{k=1}^{70} \frac{8}{(2k-1)^2 \pi^2 + 4} = 0.758699334 \quad \tanh(1) = 0.761594156$$

$$x \prod_{k=1}^{\infty} \left(1 + \frac{x^2}{k^2 \pi^2} \right) = \sinh(x) \quad \text{we will find p derivative} \quad p' \left[x \cdot \prod_{k=1}^{\infty} \left(1 + \frac{x^2}{k^2 \pi^2} \right) \right] = p'(\sinh(x)) \rightarrow e^{\frac{1}{x}} \left(\prod_{k=1}^{\infty} e^{\frac{2x}{k^2 \pi^2 + x^2}} = e^{\frac{1}{\tanh(x)}} \right) \rightarrow \frac{1}{x} + \sum_{k=1}^{\infty} \frac{2x}{k^2 \pi^2 + x^2} = \frac{1}{\tanh(x)} \quad x=4$$

$$\text{we will find L derivative} \quad L' \left[x \cdot \prod_{k=1}^{\infty} \left(1 + \frac{x^2}{k^2 \pi^2} \right) \right] = L'(\sinh(x)) \rightarrow 1 + \sum_{k=1}^{\infty} \frac{2x^2}{k^2 \pi^2 + x^2} = \frac{x}{\tanh(x)} \rightarrow \frac{1}{x} + \sum_{k=1}^{\infty} \frac{2x}{k^2 \pi^2} = \frac{1}{\tanh(x)}$$

$$0.25 + \sum_{k=1}^{70} \frac{8}{k^2 \pi^2 + 16} = 0.989175154 \quad \frac{1}{\tanh(4)} = 1.00067115$$

$$\prod_{k=0}^{\infty} (1+x2^k) = \frac{1}{1-x} \quad |x| < 1 \quad \text{we will find p derivative} \quad p' \left[\prod_{k=0}^{\infty} (1+x2^k) \right] = p' \left(\frac{1}{1-x} \right) \rightarrow \prod_{k=0}^{\infty} e^{\frac{2^k \cdot x^{2^k-1}}{1+x2^k}} = e^{\frac{1}{1-x}} \rightarrow \sum_{k=0}^{\infty} \frac{2^k x^{2^k-1}}{1+x2^k} = \frac{1}{1-x} \quad x=0.2 \quad \sum_{k=0}^{70} \frac{2^k \cdot 0.2^{2^k-1}}{1+0.2^{2^k}} = 1.25 \quad \frac{1}{1-0.2} = 1.25$$

$$\text{we will find L derivative} \quad L' \left[\prod_{k=0}^{\infty} (1+x2^k) \right] = L' \left(\frac{1}{1-x} \right) \rightarrow \sum_{k=0}^{\infty} \frac{2^k x^{2^k}}{1+x2^k} = \frac{x}{x-1} \quad x=0.2 \quad \sum_{k=0}^{70} \frac{2^k \cdot 0.2^{2^k}}{1+0.2^{2^k}} = 0.25 \quad \frac{0.2}{1-0.2} = 0.25$$

$$\prod_{k=1}^{\infty} \cos\left(\frac{x}{2^k}\right) = \frac{\sin(x)}{x} \quad \text{we will find p derivative} \quad p' \left(\prod_{k=1}^{\infty} \cos\left(\frac{x}{2^k}\right) \right) = p' \left(\frac{\sin(x)}{x} \right) \rightarrow \prod_{k=1}^{\infty} e^{\frac{-\tan\left(\frac{x}{2^k}\right)}{2^k}} = e^{\frac{1}{\tan(x)} + \frac{-1}{x}} \rightarrow \sum_{k=1}^{\infty} \frac{\tan\left(\frac{x}{2^k}\right)}{2^k} = \left(\frac{1}{x}\right) - \frac{1}{\tan(x)}$$

$$\text{we will find L derivative} \quad L' \left(\prod_{k=1}^{\infty} \cos\left(\frac{x}{2^k}\right) \right) = L' \left(\frac{\sin(x)}{x} \right) \rightarrow \sum_{k=1}^{\infty} \frac{x \cdot \tan\left(\frac{x}{2^k}\right)}{2^k} = 1 - \frac{x}{\tan(x)} \rightarrow \sum_{k=1}^{\infty} \frac{\tan\left(\frac{x}{2^k}\right)}{2^k} = \left(\frac{1}{x}\right) - \frac{1}{\tan(x)}$$

$$x = \frac{\pi}{4} \quad \sum_{k=1}^{70} \frac{\tan\left(\frac{\pi}{2^{k+2}}\right)}{2^k} = 0.273239545 \quad \frac{4}{\pi} - \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 0.273239545 \quad x = 2 \quad \sum_{k=1}^{70} \frac{\tan\left(\frac{2}{2^k}\right)}{2^k} = 0.957657554 \quad \frac{1}{2} - \frac{1}{\tan(2)} = 0.957657554$$

$$\prod_{k=1}^{\infty} \cosh\left(\frac{x}{2^k}\right) = \frac{\sinh(x)}{x}$$

we will find p derivative

$$p'\left(\prod_{k=1}^{\infty} \cosh\left(\frac{x}{2^k}\right)\right) = p'\left(\frac{\sinh(x)}{x}\right)$$

$$\rightarrow \prod_{k=1}^{\infty} e^{\frac{\tanh\left(\frac{x}{2^k}\right)}{2^k}} = e^{\frac{1}{\tanh(x)} - \frac{1}{x}} \rightarrow \sum_{k=1}^{\infty} \frac{\tanh\left(\frac{x}{2^k}\right)}{2^k} = \frac{1}{\tanh(x)} - \frac{1}{x}$$

we will find L derivative

$$L'\left(\prod_{k=1}^{\infty} \cosh\left(\frac{x}{2^k}\right)\right) = L'\left(\frac{\sinh(x)}{x}\right)$$

$$\rightarrow \left(\sum_{k=1}^{\infty} \frac{x \cdot \tanh\left(\frac{x}{2^k}\right)}{2^k}\right) = \frac{x}{\tanh(x)} - 1 \rightarrow \sum_{k=1}^{\infty} \frac{\tanh\left(\frac{x}{2^k}\right)}{2^k} = \frac{1}{\tanh(x)} - \frac{1}{x}$$

$$x = 0.5 \quad \sum_{k=1}^{70} \frac{\tanh\left(\frac{1}{2^{k+1}}\right)}{2^k} = 0.163953414 \quad \frac{1}{\tanh(0.5)} - 2 = 0.163953414$$

$$x = 0.9 \quad \sum_{k=1}^{70} \frac{\tanh\left(\frac{0.9}{2^k}\right)}{2^k} = 0.284956142 \quad \frac{1}{\tanh(0.9)} - \frac{1}{0.9} = 0.284956142$$

The system of differential equations defines parametric equations for curve in space

$$\frac{d}{dt}x(t) = a \cdot x(t) \cdot \ln(x(t) \cdot z(t)) \quad \left[P^x(t) = (x(t) \cdot z(t))^a \right] \quad a := 0.001 \quad \underline{\underline{c}} := 1.54 \quad r := 0.51$$

$$\frac{d}{dt}y(t) = (y(t) - x(t) \cdot z(t) + c \cdot x(t)) \cdot y(t)$$

$$D(t, Q) := \begin{bmatrix} a \cdot Q_0 \cdot \ln(Q_0 \cdot Q_2) \\ (Q_1 - Q_0 \cdot Q_2 + c \cdot Q_0) \cdot Q_1 \\ (Q_0 \cdot Q_1 - r \cdot Q_2) \cdot Q_2 \end{bmatrix}$$

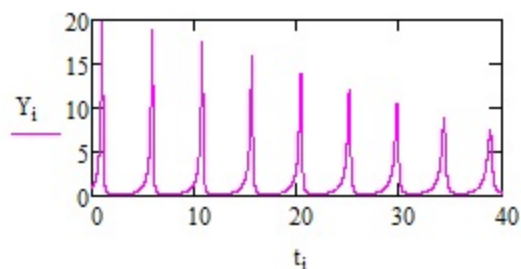
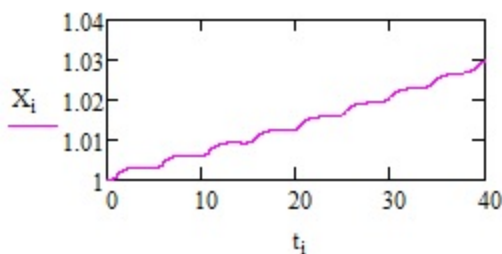
$$\frac{d}{dt}z(t) = (x(t) \cdot y(t) - r \cdot z(t)) \cdot z(t)$$

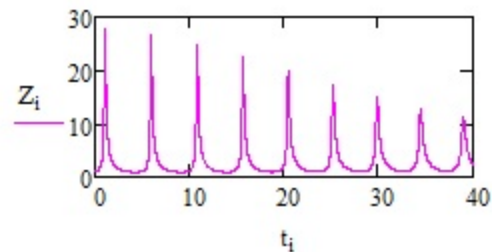
Npts := 10000

$$\underline{\underline{L}} := \text{rkfixed} \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, 0, 50, \text{Npts}, D \right)$$

Can look at the individual components as two dimensional

$$t := L^{(0)} \quad X := L^{(1)} \quad Y := L^{(2)} \quad Z := L^{(3)} \quad i := 0..Npts \quad \underline{\underline{\epsilon}} := 0.00001$$





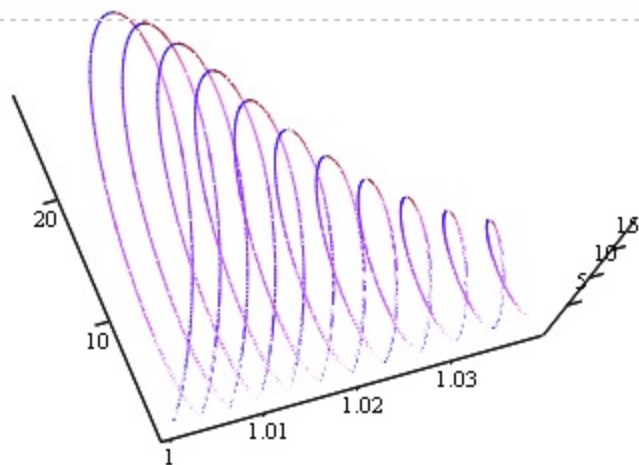
Can view this space curve as parametric surface plot

$$a = 0.001 \quad c = 1.54 \quad r = 0.51 \quad \epsilon = 0.00001$$

$$\underline{\underline{R}}^{\langle 0 \rangle} := X \quad \underline{\underline{R}}^{\langle 1 \rangle} := X + \epsilon \quad \underline{\underline{S}}^{\langle 0 \rangle} := Y \quad \underline{\underline{S}}^{\langle 1 \rangle} := Y + \epsilon$$

$$\underline{\underline{T}}^{\langle 0 \rangle} := Z \quad \underline{\underline{T}}^{\langle 1 \rangle} := Z + \epsilon$$

$$x(t) \neq 0 \quad z(t) \neq 0 \quad x(t) \cdot z(t) > 0$$



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(R, S, T)

Find the situation equilibrium of this system

$$I \quad a \cdot x(t) \cdot \ln(x(t) \cdot z(t)) = 0$$

I

$$I \quad (y(t) - x(t) \cdot z(t) + c \cdot x(t)) \cdot y(t) = 0$$

I

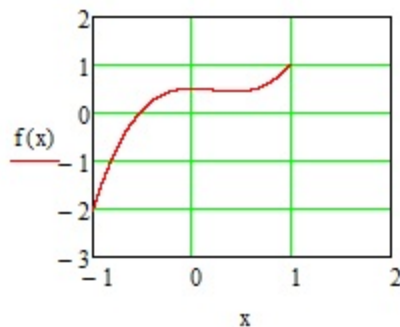
$$I \quad (x(t) \cdot y(t) - r \cdot z(t)) \cdot z(t) = 0$$

hence obtain $c \cdot x^3 - x^2 + r = 0$ $y = 1 - c \cdot x$ $z = \frac{1}{x}$ let $a = 0.001$ $c = 1.54$ $r = 0.51$

then you get the equation $f(x) := 1.54 \cdot x^3 - x^2 + 0.51$

the initial approximation of the root of the equation is $x := -0.5$ $s_{\text{root}} := \text{root}(f(x), x)$ $s = -0.52992764$

$$x_{\text{root}} := -1, -0.999.. 1$$



$$y := 1 - 1.54s \quad y = 1.81608857 \quad z := \frac{1}{s} \quad z = -1.88705007$$

Then point $A(-0.52992764, 1.81608857, -1.88705007)$ is the situation equilibrium of this system

Let $u(t) = x(t) + 0.5299$ $v(t) = y(t) - 1.8161$ $q(t) = z(t) + 1.8870$ thus

$$\frac{I}{I} u'(t) = a \cdot (u - 0.5299) \cdot \ln[(u - 0.5299) \cdot (q - 1.8870)]$$

$$\frac{I}{I} v'(t) = [(v + 1.8161) - (u - 0.5299) \cdot (q - 1.8870) + 1.54(u - 0.5299)] \cdot (v + 1.8161)$$

$$\frac{I}{I} q'(t) = [(u - 0.5299)(v + 1.8161) - 0.51(q - 1.8870)] \cdot (q - 1.8870)$$

Linearization of this system is

$$u'(t) = 0.0099992 \cdot u + 0 \cdot v + 0.0028082 \cdot q$$

$$v'(t) = 6.2237747 \cdot u + 1.8162327 \cdot v + 0.9623514 \cdot q$$

$$q'(t) = -3.4269807 \cdot u + 0.9999214 \cdot v + 0.9623886 \cdot q$$

$$M := \begin{pmatrix} 0.0099992 & 0 & 0.0028082 \\ 6.2237747 & 1.8162327 & 1.792 \\ -3.4269807 & 0.9999214 & 0.9623886 \end{pmatrix}$$

The eigenvalue of this matrix is

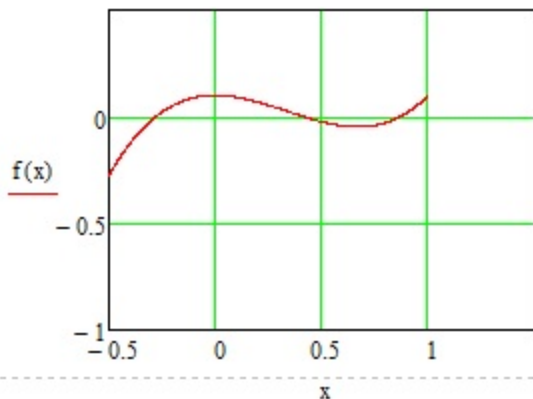
$$\text{eigenvals}(M) = \begin{pmatrix} -0.00337652 + 0.11106767i \\ -0.00337652 - 0.11106767i \\ 2.79537355 \end{pmatrix}$$

So the situation equilibrium of this system is unstable

If $a = 0.001$ $c = 1.0$ $r = 0.1$ than of this system has three situation equilibrium

$$f(x) := x^3 - x^2 + 0.1$$

$$x := 0.5 \quad s := \text{root}(f(x), x) \quad s = 0.41260557 \quad x := -1, -0.999.. 1$$



$$\begin{pmatrix} .50000000000000000000 \\ -.30901699437494742410 \\ .80901699437494742410 \end{pmatrix}$$

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$$\frac{d}{dt}x(t) = a \cdot x(t) \cdot \ln(x(t) \cdot z(t))$$

$$[P^x(t) = (x(t) \cdot z(t))^a]$$

$$a := 0.001 \quad c := 1 \quad r := 0.1$$

$$\frac{d}{dt}y(t) = (y(t) - x(t) \cdot z(t) + c \cdot x(t)) \cdot y(t)$$

$$D(t, Q) := \begin{bmatrix} a \cdot Q_0 \cdot \ln(Q_0 \cdot Q_2) \\ (Q_1 - Q_0 \cdot Q_2 + c \cdot Q_0) \cdot Q_1 \\ (Q_0 \cdot Q_1 - r \cdot Q_2) \cdot Q_2 \end{bmatrix}$$

$$\frac{d}{dt}z(t) = (x(t) \cdot y(t) - r \cdot z(t)) \cdot z(t)$$

$$N_{pts} := 10000$$

$$L := rkfixed \left[\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, 0, 50, N_{pts}, D \right]$$

Can look at the individual components as two dimensional

$$t := L^{(0)}$$

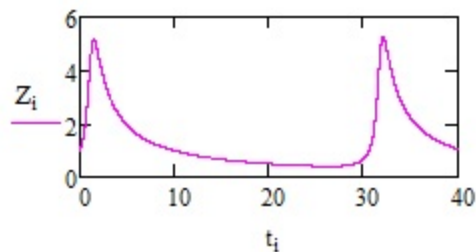
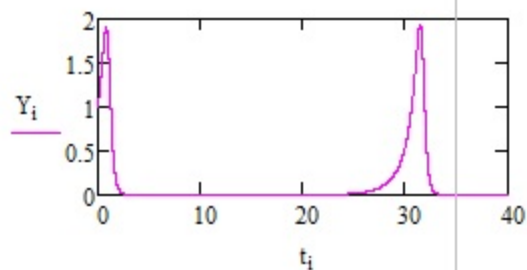
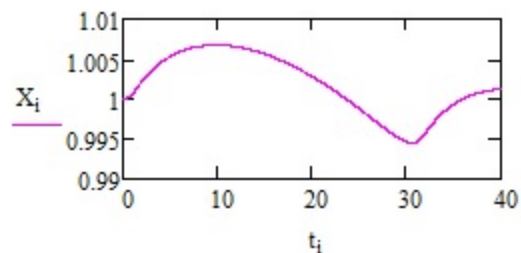
$$X := L^{(1)}$$

$$Y := L^{(2)}$$

$$Z := L^{(3)}$$

$$i := 0..N_{pts}$$

$$\epsilon := 0.00001$$



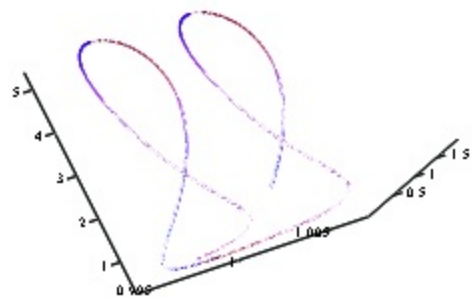
Can view this space curve as parametric surface plot

$a = 0.001$ $c = 1.0$ $r = 0.1$ $\epsilon = 0.00001$

$R^{(0)} := X$ $R^{(1)} := X + \epsilon$ $S^{(0)} := Y$ $S^{(1)} := Y + \epsilon$

$T^{(0)} := Z$ $T^{(1)} := Z + \epsilon$

$x(t) \neq 0$ $z(t) \neq 0$ $x(t) \cdot z(t) > 0$



(R, S, T)

ATTRACTOR

The system of differential equations defines parametric equations for curve in space

$a := 0.01$ $c := 1$ $r := 2$ $Npts := 500$ $x(t) > 0$ $y(t) > 0$ $z(t) > 0$ $a \neq 1$

$$\frac{d}{dt}x(t) = x(t) \ln\left(\frac{x(t)^a}{y(t) + z(t)}\right)$$

$$p'x(t) = \frac{x(t)^a}{y(t) + z(t)}$$

$$\frac{d}{dt}y(t) = y(t) \cdot \ln\left(\frac{c \cdot z(t)}{x(t) \cdot y(t)}\right)$$

$$p'y(t) = \frac{c \cdot z(t)}{x(t) + y(t)}$$

$$\frac{d}{dt}z(t) = z(t) \cdot \ln\left(\frac{r \cdot x(t)}{y(t) + z(t)}\right)$$

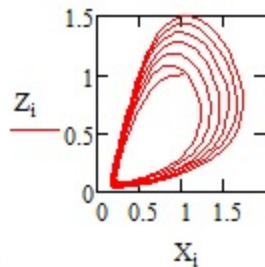
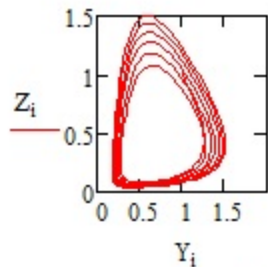
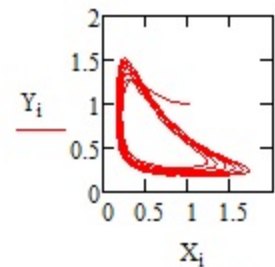
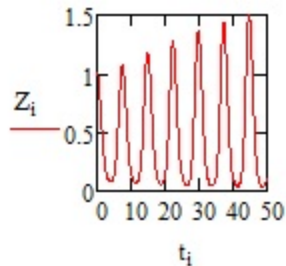
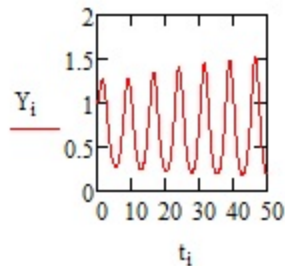
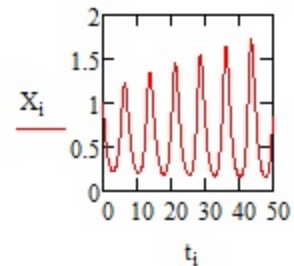
$$p'z(t) = \frac{r \cdot x(t)}{y(t) + z(t)}$$

$$D(t, Q) := \begin{bmatrix} Q_0 \cdot \ln\left[\frac{(Q_0)^a}{Q_1 + Q_2}\right] \\ Q_1 \cdot \ln\left[\frac{c \cdot Q_2}{Q_0 \cdot Q_1}\right] \\ Q_2 \cdot \ln\left[\frac{r \cdot Q_0}{Q_1 + Q_2}\right] \end{bmatrix}$$

$$\underline{L} := \text{rkfixed} \left[\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, 0, 50, Npts, D \right]$$

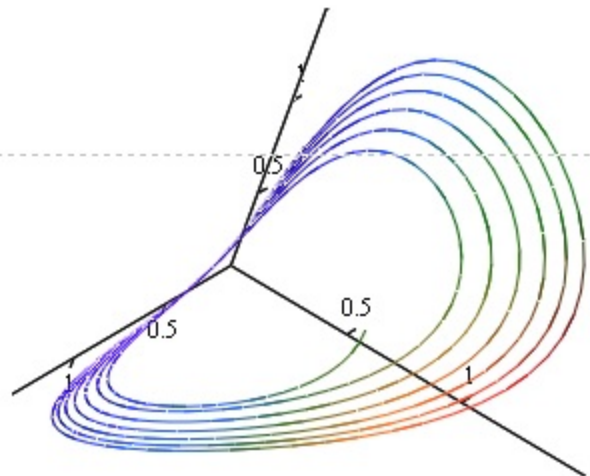
Can look at the individual components as two dimensional

$t := L^{(0)}$ $X := L^{(1)}$ $Y := L^{(2)}$ $Z := L^{(3)}$ $i := 0..Npts$ $\underline{\epsilon} := 0.0001$



Can view this space curve as parametric surface plot

$$\underline{\underline{R}}^{(0)} := X \quad \underline{\underline{R}}^{(1)} := X + \epsilon \quad \underline{\underline{S}}^{(0)} := Y \quad \underline{\underline{S}}^{(1)} := Y + \epsilon \quad \underline{\underline{T}}^{(0)} := Z \quad \underline{\underline{T}}^{(1)} := Z + \epsilon$$



(R, S, T)

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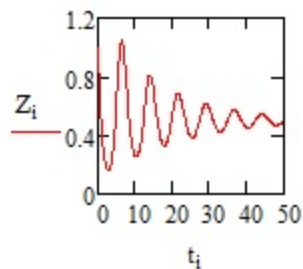
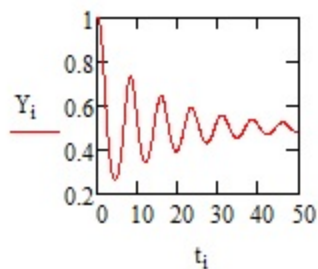
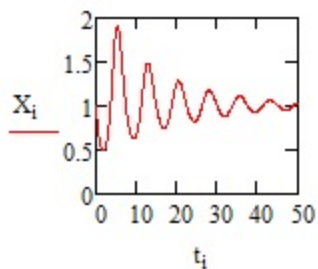
$$\underline{a} := 0.1 \quad \underline{c} := 1 \quad \underline{r} := 1$$

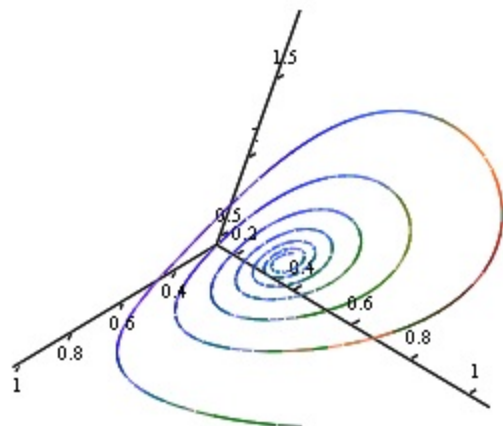
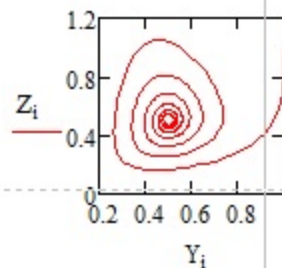
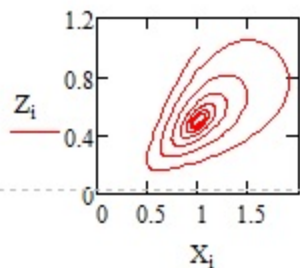
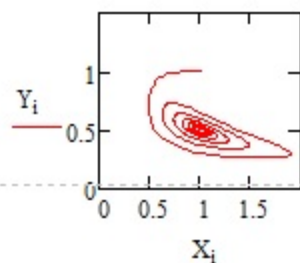
$$D(t, Q) := \begin{bmatrix} Q_0 \cdot \ln \left[\frac{(Q_0)^a}{Q_1 + Q_2} \right] \\ Q_1 \cdot \ln \left(\frac{c \cdot Q_2}{Q_0 \cdot Q_1} \right) \\ Q_2 \cdot \ln \left(\frac{r \cdot Q_0}{Q_1 + Q_2} \right) \end{bmatrix}$$

$$\underline{L} := \text{rkfixed} \left[\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, 0, 50, \text{Npts}, D \right]$$

$$t := L^{(0)} \quad X := L^{(1)} \quad Y := L^{(2)} \quad Z := L^{(3)} \quad i := 0..Npts \quad \underline{\epsilon} := 0.00001$$

$$R^{(0)} := X \quad R^{(1)} := X + \epsilon \quad S^{(0)} := Y \quad S^{(1)} := Y + \epsilon \quad T^{(0)} := Z \quad T^{(1)} := Z + \epsilon$$





(R, S, T)

$$\underline{\underline{a}} := 0.01 \quad \underline{\underline{c}} := 10 \quad \underline{\underline{r}} := 1$$

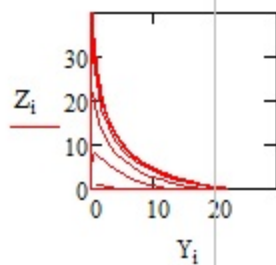
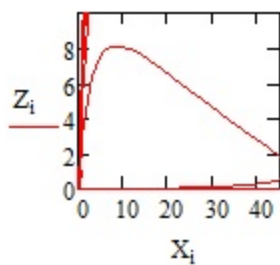
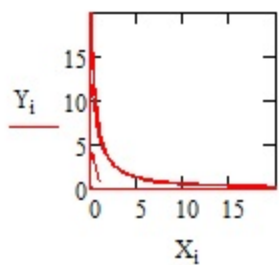
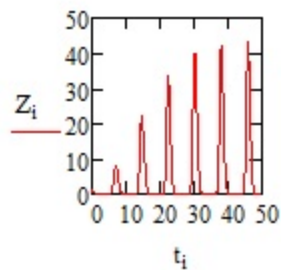
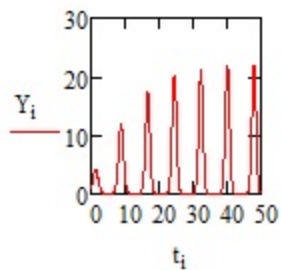
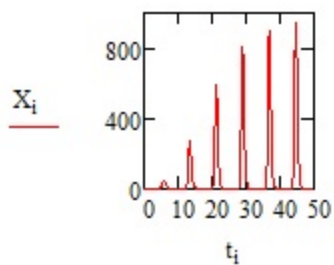
$$\underline{\underline{D}}(t, Q) := \begin{bmatrix} Q_0 \cdot \ln \left[\frac{(Q_0)^a}{Q_1 + Q_2} \right] \\ Q_1 \cdot \ln \left[\frac{c \cdot Q_2}{Q_0 \cdot Q_1} \right] \\ Q_2 \cdot \ln \left[\frac{r \cdot Q_0}{Q_1 + Q_2} \right] \end{bmatrix}$$

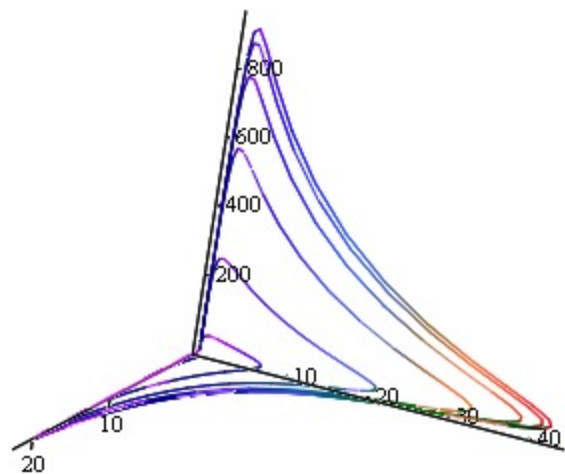
$$\underline{\underline{I}} := \text{rkfixed} \left[\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, 0, 50, \text{Npts}, D \right]$$

+

$t := L^{(0)}$ $X := L^{(1)}$ $Y := L^{(2)}$ $Z := L^{(3)}$ $i := 0..Npts$ $\underline{\underline{\epsilon}} := 0.0001$

$R^{(0)} := X$ $R^{(1)} := X + \epsilon$ $S^{(0)} := Y$ $S^{(1)} := Y + \epsilon$ $T^{(0)} := Z$ $T^{(1)} := Z + \epsilon$





Find the situation equilibrium of this system

$$x \cdot \ln\left(\frac{x^a}{y+z}\right) = 0$$

$$y \cdot \ln\left(\frac{c \cdot z}{x \cdot y}\right) = 0 \quad \Rightarrow$$

$$z \cdot \ln\left(\frac{r \cdot x}{y+z}\right) = 0$$

+

(R, S, T)

$$x = r^{\frac{1}{a-1}} \quad y = \frac{c \cdot r^{\frac{a}{a-1}}}{\frac{1}{a-1}} \quad z = \frac{r^{\frac{a+1}{a-1}}}{\frac{1}{a-1}} \quad \text{If } c=1 \quad r=1 \text{ then } x=1 \quad y=0.5 \quad z=0.5$$

$$u(t) = x(t) - 1 \quad v(t) = y(t) - 0.5 \quad q = z(t) - 0.5 \quad A(x_0, y_0, z_0) \leftrightarrow A_1(u_0, v_0, q_0) = A_1(0, 0, 0)$$

$$u'(t) = (u + 1) \cdot \ln \left[\frac{(u + 1)^a}{v + q + 1} \right]$$

Linearization of this system is

$$u'(t) = a \cdot u - a \cdot v - a \cdot q$$

$$v'(t) = (v + 0.5) \cdot \ln \left[\frac{q + 0.5}{(u + 1) \cdot (v + 0.5)} \right]$$

$$v'(t) = -0.5u - v + q$$

$$q'(t) = (q + 0.5) \cdot \ln \left(\frac{u + 1}{v + q + 1} \right)$$

$$q'(t) = 0.5u - 0.5v - 0.5q$$

Find the eigenvalue μ of this system

$$\begin{pmatrix} a - \mu & -a & -a \\ -0.5 & -1 - \mu & 1 \\ 0.5 & -0.5 & -0.5 - \mu \end{pmatrix} = 0 \Rightarrow \mu \cdot (\mu^2 + \mu(1.5 - a) - 1.5a + 1) = 0$$

$$\mu_1 = 0 \quad \mu_{2,3} = \frac{\pm \sqrt{a^2 + 3a - \frac{7}{4}} + a - 1.5}{2} \quad \text{If } a > \frac{2}{3} \text{ the root } \mu_2 = \frac{\sqrt{a^2 + 3a - \frac{7}{4}} + a - 1.5}{2} > 0$$

There is the bifurcation point at $a = 2/3$ If $a=0.5$ then

$$M := \begin{pmatrix} a & -a & -a \\ -0.5 & -1 & 1 \\ 0.5 & -0.5 & -0.5 \end{pmatrix} \quad |M| = 0$$

The eigenvalue of this matrix is

$$\text{eigenvals}(M) = \begin{pmatrix} 0 \\ -0.745 + 0.656i \\ -0.745 - 0.656i \end{pmatrix}$$

Hence the situation equilibrium(1, 0.5, 0.5) of this system is stable(for $c = 1, r = 1$)

If $a=1$ then the situation equilibrium of this system is

$$x = \gamma \quad y = \frac{c \cdot \gamma}{\gamma^2} \quad z = \frac{\gamma^2}{\gamma^2} \quad \text{where } \gamma > 0 \text{ is an arbitrary number}$$

Referens . In wikipedia , in the article multiplicative there are definition of several forms of derivatives .

The anaquadrat derivative $[Df_{\circ}](h) = [D\bar{f}](\bar{h}^2)$.

The biguadrat derivative $[Df_{\circ}^{\circ}](h) = [[D\bar{f}](\bar{h})]^{\frac{1}{2}}$.

The harmonic derivative $[Df^{\circ}](h) = \frac{1}{[D(1//f)](h)}$, where $1//f = \begin{cases} \frac{1}{x} , & \text{if } x \neq 0 \\ 0 , & \text{if } x = 0 \end{cases}$.

The anaharmon derivative $[Df_{\circ}](a) = [D\bar{f}](1//a)$.

The anageomet derivative $D^*(f) = \lim_{x \rightarrow h} \frac{f(x) - f(h)}{\ln(x) - \ln(h)}$.

The quadrat derivative $[Df^{\circ}](h) = [[D(\bar{f}^2)](h)]^{\frac{1}{2}}$, where $x^{\frac{1}{2}} = \begin{cases} \sqrt{x} , & \text{if } x \geq 0 \\ -\sqrt{-x} , & \text{if } x < 0 \end{cases}$, $\bar{x}^2 = \begin{cases} x^2 , & \text{if } x \geq 0 \\ -x^2 , & \text{if } x < 0 \end{cases}$.

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