Fan-Beam Tomography Iterative Algorithm Based on Fourier Transform

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***Abstract*—In tomography with a small number of views (i.e., optical tomography) there is a big demand for the algorithms able to adapt and efficiently use different types of a priori information about the solution. The Gerchberg-Papoulis algorithm (G-P) is known as one of the best iterative methods of a limited data tomography with parallel arrangement of projections because of its wide range of constraints it is capable to embed into processing steps. The G-P algorithm is based on the theorem of the central slice in Fourier space for parallel geometry of scanning and intensive interpolation in both image and frequency domains. In fan-beam geometry such algorithm has not been investigated, because of the central slice theorem lacking. In this paper recently developed theorem of the central slice is stated for fan-beam geometry, and on its basis the new iterative G-P algorithm is developed.**

I. INTRODUCTION

In tomography with a small number of views there is a big demand for the algorithms able to adapt and efficiently use different types of a priori information about the solution. One of the well developed reconstruction algorithms known for its adaptivity to a priori constraints for parallel geometry is Gerchberg-Papoulis algorithm (G-P), comprehensively investigated, for example in [1] - [3]. It uses alternately iterations between image space and its Fourier transform. In papers [4], [5] the central slice theorem which connects Fourier transform of fan-beam projections with Fourier-image of object has been developed. On the basis of this theorem, fan-beam G-P algorithm in the same papers is offered and schematically described (without numerical results). In this paper, the first numerical results for fan-beam iterative algorithm are shown.

II. THEORY

In this section we state the central slice theorem in the case of a fan-beam tomography (details of proof can be found in our recent work [5]). The essence of our approach is the nonlinear coordinate transformation developing a problem of a fan-beam tomography into a problem of a parallel projection tomography, where it is formulated in terms of new space deformed in a special way.

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The fan-beam projections registered in original space become parallel in the new deformed space. Therefore, once the central slice theorem is valid for geometry of parallel projections, it can be applied to the deformed versions of tomogram too. Each view needs its own image deformation, so that we need to correct each deformation separately, in its own way. After processing of a single projection within this deformed space, it is necessary to perform the inverse deformation transform mapping the tomogram back onto original space, before the next projection processing.

Let us consider the standard scheme of a two-dimensional fan-beam tomography (see Fig.1) we use in the derivation of fan-beam analogue of the central slice theorem [5].

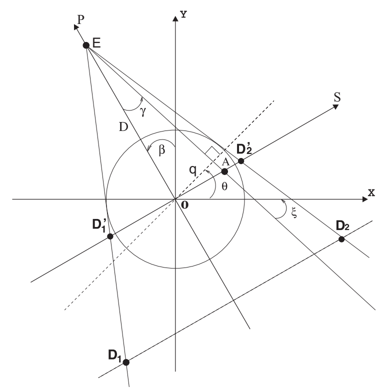

Fig. 1. Projection formation on the flat detector in the scheme of fan-beam tomography.

Here the support of unknown function $g(x, y)$ is contained in a unit circle $x^2 + y^2 < 1$. The sources of fan beams E are located on a circle of radius D, and detectors are located on a straight line D_1D_2 (see Fig. 1). Each fan is parameterized by angle β and each beam EA in the fan can be characterised by an angle γ with the central beam EO, and distance q from the beam to the centre of coordinates O. The angle θ is an angle between an axis x and a normal to the beam, and the angle between the beam and the axis x is denoted by ξ. The signal is measured on a line of detectors D_1D_2. We use equivalent arrangement of signal registration in form of virtual detector on a straight line $D_1^{'}D_2^{'}$.

Let us apply the following projective ("deforming") transform mapping of the rotated system of coordinates (s, p) onto another system (u, v):

$$\begin{cases} u = s/(1 - \frac{p}{D}) = s/Q; \\ v = p. \end{cases} \tag{1}$$

Under this transform, the system of fan-beams in coordinates (x, y) becomes a system of the straight lines parallel to the coordinate axis v. The initial image $g(x, y)$ undergoes deformation too, and become a new image $g(X(u, v), Y(u, v))$ depending on the angle β. Let us denote inverse mapping as $u = U(x, y), v = V(x, y)$. We introduce (r, ϕ) - polar coordinates of a point (x, y), and (q, θ) - normal coordinates of the fan beam which is passing through this point (Fig. 1). Then after some algebra (details can be found in [3], [6]), the fan-beam projection can be expressed as follows:

$$f_\beta(u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(q - U(x, y))g(x, y)dxdy. \quad (2)$$

It can be shown that equation (2) in new coordinates can be written as:

$$\begin{aligned} f_\beta(u) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |Q|\delta(q - U)g(u, v)dudv \\ &= \frac{1}{\cos\gamma(u)}\int_{-\infty}^{\infty} g(X(u, v), Y(u, v))dv. \quad (3) \end{aligned}$$

We denote integrand $g(u, v)$ in integral (3) as $h_\beta(u, v)$ emphasizing the dependencies of coordinates (u, v) upon β and introduce new function $f'_\beta(u) = f_\beta(u)\cos\gamma(u)$. Applying the Fourier transform to function $f'_\beta(u)$, we receive the central slice theorem for fan-beam projections [4], [5]:

$$\begin{aligned} \tilde{f'}_\beta(\nu_u) &= \int \exp(-2\pi i\nu_u u)du \int h_\beta(u, v)dv \\ &= \tilde{h}_\beta(\nu_u, \nu_v)|_{\nu_v=0}. \quad (4) \end{aligned}$$

We can summarize that the new central slice theorem is expressed in terms of coordinates (u, v) received from initial coordinates by nonlinear projective transformation of variables (1). Hence, the Fourier transform of the modified fan-beam projection (i.e. multiplied by cosine of incidence angle of each beam in a fan-beam), coincides with the central section of two-dimensional Fourier transform of the deformed object $h_\beta(u, v)$.

Iterative G-P algorithm essentially use the central slice theorem. This method possess many attractive features (e.g., superresolution) useful in image reconstruction from limited data although it is computer resources demanding technique. We will present this algorithm according to paper [7] in the form:

$$\begin{aligned} g^{(n)}(x, y) &= \Phi_s^{(n)} F_2^{-1}\tilde{g}^{(n-1)}(\nu_x, \nu_y), \\ \tilde{g}^{(n)}(\nu_x, \nu_y) &= \Phi_f^{(n-1)} F_2 g^{(n-1)}(x, y), \\ g^{(0)}(x, y) &= 0,\ n = 1, 2, ..., \quad (5) \end{aligned}$$

where F_2 and F_2^{-1} are the operators of the direct and inverse two-dimensional Fourier-transform, respectively; $\Phi_s^{(n)}, \Phi_f^{(n)}$ are operators correcting the n-th iterative solution and its Fourier-image, respectively. The latter operators include a priori information: $\Phi_s^{(n)}$ operates in the field of the tomogram, $\Phi_f^{(n)}$ operates in Fourier-space. In particular, $\Phi_f^{(n)}$ is responsible for transferring the Fourier-spectrum of the projections from radial lines in the tomogram's Fourier-space onto the entire two-dimensional Fourier-space. This procedure is the central part of the G-P algorithm, which takes into account all measured data, i.e. the projections.

III. Numerical simulations

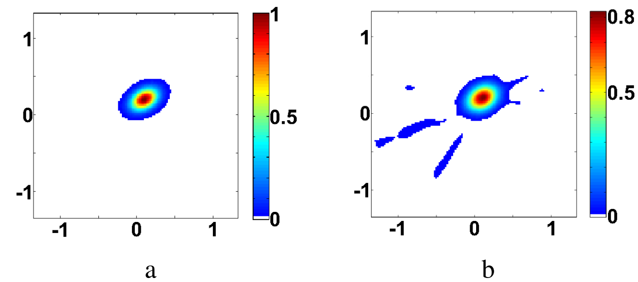

Fig. 2. a) Original tomogram; b) result of reconstruction by G-P algorithm with parameters: 13 projections, 128 rays each, D = 1.5, image size 128 x 128, 20 iterations; reconstruction error $RMS = 10\%$.

The results received by means of the G-P algorithm, are presented in a Fig. 2 where as an elementary phantom the model of shifted and rotated gaussian is considered. Comprehensive numerical studies are to be presented at the conference.

IV. Conclusion

In this work the characteristics of a version of iterative Gerchberg-Papoulis algorithm for fan-beam scheme in two-dimensional tomography are investigated.

The algorithm proposed is based on the new theorem of connection between Fourier-transform of projections and radial sections of two-dimensional Fourier-transform of deformed tomogram. Fan-beam version of G-P algorithm for small number of views has proved as a time consuming method which, however, practically does not concede in accuracy to a parallel geometry case. The best results of reconstruction are received for the fan-beam G-P algorithm with bicubic interpolation. When dealing with small number of noisy projections, their smoothing is necessary at noise level over 5%, and on smaller noise the iterative algorithm itself possesses the regularization properties.

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