Influence of the Dzyaloshinskii-Moriya interaction on vortex states in magnetic nanodisks

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Abstract. Curling magnetic vortices with chiral rotation sense and polarity of the core magnetization can exist in magnetic circular nanostructures. Broken mirror symmetry at surfaces/interfaces of magnetic nanostructures induces chiral Dzyaloshinskii-Moriya interactions which may strongly affect the magnetic properties. Using a micromagnetic approach, we investigate the influence of these chiral interactions on the vortex states in magnetic nanodisks. We calculate numerically the shape, size, and stability of the vortices in equilibrium as functions of magnetic field and the material and geometrical parameters. As a result, under the influence of the chiral magnetic interactions vortices of opposite chirality should have different sizes. We provide detailed numerical analysis of this effect, which can be applied to measure the strength of the induced Dzyaloshinskii-Moriya coupling in magnetic thin film elements.

1. Introduction

In thin magnetic films and multilayers chiral symmetry breaking at surfaces and interfaces induces specific *chiral* (so-called Dzyaloshinskii-Moriya(DM)) interactions [1, 2]. They can stabilize modulated and localized states with a fixed rotation sense of the magnetization [2]. Recently such induced chiral modulations have been discovered in magnetic nanolayers (see e.g. [3]). Chiral effects also have been observed in vortex states of magnetic nanodisks [4]. Vortex states in thin magnetic nanodots are intensively investigated and considered as promising candidate for high-density recording media [5, 6]. In this contribution we investigate, within a phenomenological micromagnetic theory, the vortex states in magnetic nanodots with induced DM interactions. In our paper we demonstrate that the induced DM interaction strongly influences the equilibrium states of the vortices and allows to control the chirality of vortices and values of the magnetization flux associated with the vortex core.

2. Numerical solutions

The equilibrium parameters of vortex states in a circular nanodisk are derived by minimization of a micromagnetic energy for a uniaxial ferromagnet with DM interactions [2]. The energy density of this system can be written in the following standard form [2, 7]

$$w = A \sum_{i,j} \left(\frac{dm_j}{dx_i} \right)^2 + K(\mathbf{m} \cdot \mathbf{a})^2 - \mathbf{M} \cdot \mathbf{H} - D(L_{zx}^{(y)} - L_{zy}^{(x)}), \tag{1}$$

where A is the exchange stiffness, $\mathbf{m}$ is the unity vector along the magnetization $\mathbf{M} = M_s\mathbf{m}$, M_s is the saturation magnetization, $\mathbf{a}$ is the anisotropy axis taken perpendicularly to the film surface, $\mathbf{H}$ is the applied field. In the limit of thin films the magnetodipole energy has a local character and reduces to so-called shape anisotropy $K_m = 2\pi M_s^2$ (for details see [7]). Then the "effective" uniaxial anisotropy constant can be written as $K = K_u + K_m$ (where K_u is the intrinsic uniaxial anisotropy) and should be positive to stabilize vortex states ($K > 0$). The DM energy with the Dzyaloshinskii constant D consists of invariants linear in first derivatives of the magnetization $L_{ij}^{(k)} = m_i\partial_k m_j - m_j\partial_k m_i$ (so-called Lifshitz invariants) [8]. The functional form of this energy is determined by the character of symmetry breaking at the surface/interfaces [1]. In (1) the chosen form of the DM energy favours curling modes (Fig. 1 (a), Inset). We consider vortex states in a disk of radius R_d with axisymmetric distributions of the magnetization and write the magnetization vector $\mathbf{m}$ in terms of spherical coordinates and the spatial variables in cylindrical coordinates: $\mathbf{m} = (\sin\theta\cos\psi; \sin\theta\sin\psi; \cos\theta)$, $\mathbf{r} = (\rho\cos\varphi; \rho\sin\varphi; z)$.

The variational problem for functional (1) in the magnetic field perpendicular to the surface of the nanodisk has rotationally symmetric solutions $\psi = \varphi - \pi/2, \theta = \theta(\rho)$ where the polar angle θ is derived from the equation

$$A\left(\frac{d^2\theta}{d\rho^2} + \frac{1}{\rho}\frac{d\theta}{d\rho} - \frac{1}{\rho^2}\sin\theta\cos\theta\right) + K\sin\theta\cos\theta - \frac{H}{2}\sin\theta - \frac{D}{\rho}\sin^2\theta = 0, \qquad (2)$$

with the boundary conditions $\theta(0) = 0$, $d\theta/d\rho|_{\rho=R_d} = 0$. This *free* boundary condition corresponds to zero surface energy at the disk edge $\rho = R_d$. We introduce characteristic parameters

$$l_e = \sqrt{A/K}, \quad D_0 = \sqrt{AK}, \quad \theta_h = \arccos(H/H_a), \quad H_a = 2K/M_s, \qquad (3)$$

where the *exchange* length l_e determines the size of the vortex core, D_0 is the threshold Dzyaloshinskii constant [8], θ_h is the polar angle of the magnetization in homogeneously magnetized layers, and the anisotropy field H_a equals the applied field saturating the homogeneous state ($\theta_h = 0$). The solutions of Eq. (2) for $R_d = 30\,l_e$ demonstrate a strong localization of the vortex core. At large distances from the center the angle θ approaches θ_h (Fig. 1). In Eq. (2) the DM coupling with positive D favours a rotation with an increasing angle θ (i.e. $d\theta/d\rho > 0$). As a result the vortex cores gradually widen with increasing $D > 0$. The DM interaction with negative D increases the energy for vortices with $d\theta/d\rho > 0$. In this case the solutions of Eq. (2) consist of two parts: initially the angle θ increases (unfavourable rotation) until a certain reversal angle $\theta_r > \theta_h$, and then slowly decreases (favourable rotation) approaching the limiting value θ_h (Fig. 1 (b)). Such profiles manifest a change of the rotation sense from positive slope $d\theta/d\rho > 0$ in the vortex core ($\theta < \theta_r$) to negative slope outside the core ($\theta > \theta_r$). When $|D|$ exceeds a certain critical value $|D_c|$ these solutions become unstable, and only vortices with a favourable chirality can exist.

The effective size of the vortex core can be introduced as

$$R_0 = \theta_h(d\theta/d\rho)_{\rho=0}^{-1} \qquad (4)$$

(see Inset in Fig. 1 (a)). Functions $R_0(H/H_a)$ and $R_0(D/D_0)$ plotted in Fig. 2 show that the vortex core sizes vary in a broad range under the influence of the applied field and DM coupling. The total perpendicular magnetization of the vortex core $M_c = 2\pi\int_0^{R_0} m_z\rho\,d\rho$ also strongly depends on the of DM coupling (Fig. 2 (b)).

3. Analytical results for the linear ansatz

The localized profiles in Fig. 1 can be approximated by a linear ansatz (Inset Fig. 1 (a))

$$\theta = \theta_h(\rho/R) \text{ for } 0 < \rho < R \text{ and } \theta = \theta_h \text{ for } R < \rho < R_d. \qquad (5)$$
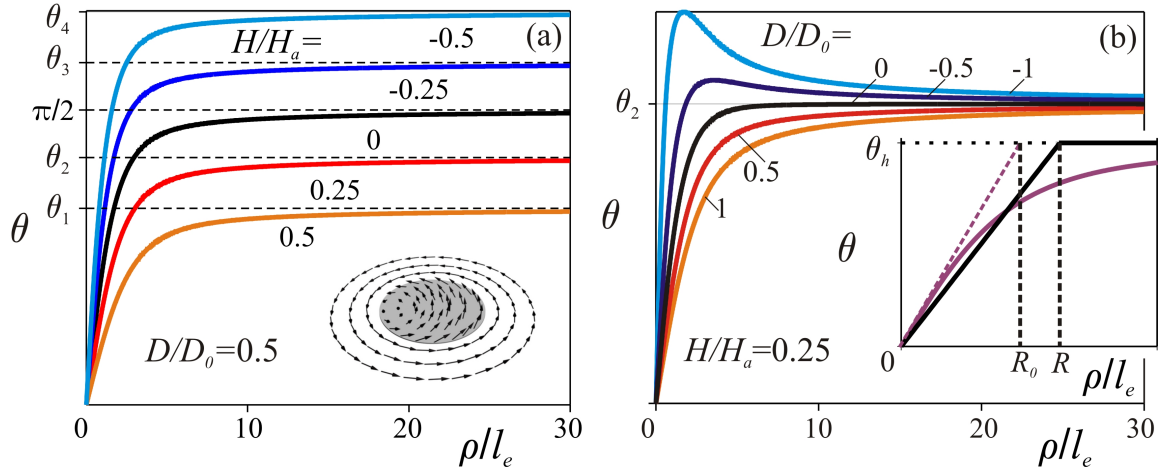
Figure 1. Magnetization profiles $\theta(\rho/l_e)$ for different values of the applied field H/H_a and for a fixed value of the Dzyaloshinskii constant $D/D_0 = 0.5$ (a). Dashed lines indicate values $\theta_h = \arccos(H/H_a)$ reached at large distances from the vortex core: $\theta_1 = \pi/3$, $\theta_2 = 1.318$, $\theta_3 = 1.823$ and $\theta_4 = 2\pi/3$. Inset shows a central part of a circular magnetic nanodisk with axisymmetric distribution of the magnetization ($\psi = \varphi - \pi/2, \theta = \theta(\rho)$). The vortex core is indicated by the shaded area. Typical vortex profiles $\theta(\rho/l_e)$ for a fixed magnetic field $H/H_a = 0.25$ and different values of the Dzyaloshinskii constant D/D_0 (b). For negative D the magnetization changes the rotation sense outside the vortex core. Inset introduces the effective size of the vortex core R_0 (Eq. (4)) and the linear ansatz for the magnetization distribution in the vortex core (Eq. (5)).

With $\theta(\rho)$ (5) the vortex energy functional $E = \int_0^{R_d} w(\theta)\rho d\rho$ at zero field is reduced to the following function of the parameter R

$$\mathcal{E}(R) = K(\pi/2 - 2/\pi)R^2 - 2\pi A \ln R \pm 2\left(1 + \pi^2/4\right)|D|\,R \tag{6}$$

and has a minimum at the point

$$R = a\,l_e\left[\sqrt{(D/D_0)^2 + b^2} \pm D/D_0\right], \tag{7}$$

where $a = (\pi/4)(\pi^2 + 4)/(\pi^2 - 4) = 1.856$, $b = 4\sqrt{\pi^2 - 4}/(4 + \pi^2) = 0.699$. In the applied magnetic field $\mathcal{E}(R)$ (6) retains its functional form, however, the coefficients in (6) become functions of H.

The simplified model (6) offers an important insight into competing energetics of the vortices. In Eq. (6) the first term $\propto R^2$ represents uniaxial anisotropy of the core. The second term ($\propto -\ln(R)$) is the exchange energy of the in-plane part of the vortex. Remarkably, the exchange energy of the vortex core does not depend on the core size. This is a general property of two-dimensional axisymmetric spin-configurations [10]. For $D = 0$ the competition between the in-plane exchange energy tending to extend the core and the uniaxial anisotropy favouring its shrinking determines the equilibrium size of the core, $R \propto \sqrt{A/K} = l_e$ (7). The DM interaction violates chiral symmetry of the solutions: it extends sizes of the vortices with favourable rotation sense and compresses the vortices with opposite chirality (Fig. 1 (b)). The difference between core sizes in vortices with different chirality $\Delta R = |R_1 - R_2| = 2\,l_e\,a\,|D|\,/D_0$ is proportional to the Dzyaloshinskii constant $|D|$. This result offers a simple method for experimental determination of the induced Dzyaloshinskii coupling in thin films.
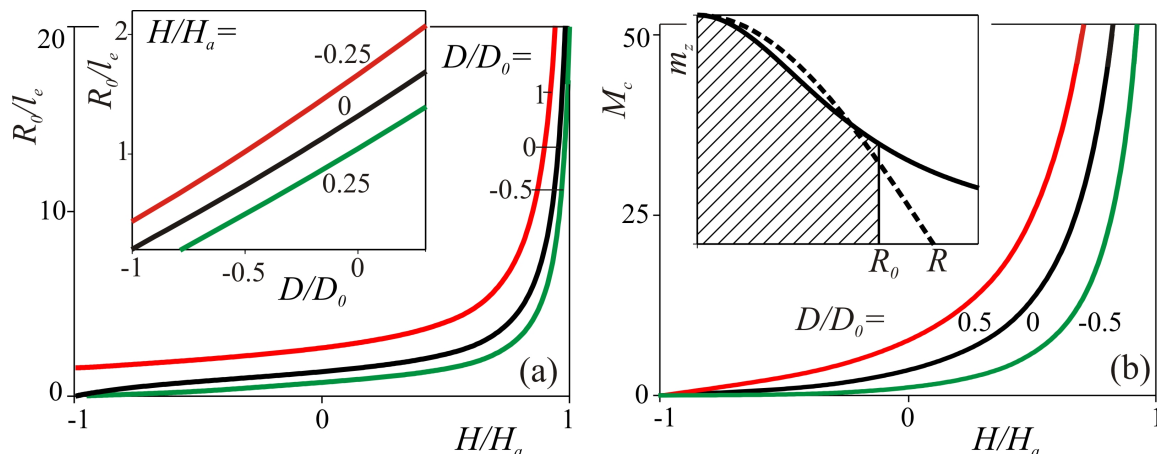
Figure 2. Effective size of vortex cores R_0 as a function of the applied field for different values of D (a). Inset shows functions $R_0(D/D_0)$ for different values of the applied fields. The total magnetization of the vortex core M_z as a function of the applied field for different values of D (b). Inset presents a typical magnetization profile $m_z(\rho)$, the magnetization distributions for the linear ansatz (dashed line) and the core magnetization (hatched area).

4. Conclusions

Theoretical analysis within a micromagnetic approach reveals a strong influence of the surface/interface induced DM interactions on the equilibrium parameters of vortex states in magnetic nanodisks. By numerically solving the differential equation (2) we have calculated the structure of the vortices and derive their sizes and magnetization as a function of the applied magnetic field and strength of DM coupling. A linear ansatz for magnetization profiles (5) allows to reduce the vortex energy to a simple functional form (6) and derive the vortex parameters in analytical form. The induced DM interactions violate chiral symmetry of the vortex states. As a result in vortices with opposite rotation sense the core should have different sizes. This effect provides an effective method for precision measurements of the induced DM coupling constant. The structure of vortices with an alteration of the rotation sense under influence of DM interactions presents another remarkable phenomenon that should be observable in experiments by switching the chirality of the vortex states in magnetic thin nanodisks.

5. References

[1] Crépieux A and Lacroix C 1998 *J. Magn. Magn. Matter* **182** 341
[2] Bogdanov A N and Rößler U K 2001 *Phys. Rev. Lett.* **87** 037203
[3] Bode M et al. 2007 *Nature* **447** 190
[4] Curcic M et al. 2008 *Phys. Rev. Lett.* **101** 197204
[5] Bohlens S, Krüger B, Dreus A, Bolte M, Meier G and Pfannkuche D 2008 *Appl. Phys. Lett.* **93** 142508
[6] Wachowiak A, Wiebe J, Bode M, Pietzsch O, Morgenstern M and Wiesendanger R 2002 *Science* **298** 577
[7] Hubert A and Schäfer R 1998 *Magnetic domains: the analysis of magnetic microstructure* (Berlin: Springer)
[8] Dzyaloshinskii I E 1964 *Soviet Physics JETP* **19** 960
[9] Guslienko K Yu 2008 *J. Nanoscience and Nanotechnology* **8** 2745
[10] Rößler U K, Bogdanov A N and Pfleiderer C 2006 *Nature* **442** 797