REMARKABLE PROPERTIES OF CYCLOID

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Imagine two random points A and B in the Earth's gravitational field (Figure 1). They can be connected by an infinite number of curves 1, 2, 3, etc. Let each curve be a chute where a body which was let in the point A is moving without initial velocity because of the force of gravity affecting, e.g. a bicyclist is riding down without pedalling or a skier is sliding. It is clear that the time of motion spent from A to B depends on the trajectory of motion and it is evident that there is such a trajectory for which it is minimum. At the first blush it seems that the straight chute is the quickest way to get from point A to point B. However, that is wrong: research has shown that there is such a curve 3 the time of moving down which is minimum. This curve of the quickest moving down is called a brachistochrone: *brachistos* — the shortest + *chronos* — time. It appears that the brachistochrone is a cycloid — a curve plotted by a point of circumference which is moving along the straight line without slipping.

The cycloidal curve has one more remarkable property: if two bodies are let free without the initial velocity at different points of cycloidal chute, they would arrive at its lowest point simultaneously as well. That's why the cycloid is called tautochrone: *tauto* — the same + *chronos* — time, that is a curve the time of motion along which does not depend on the start point.

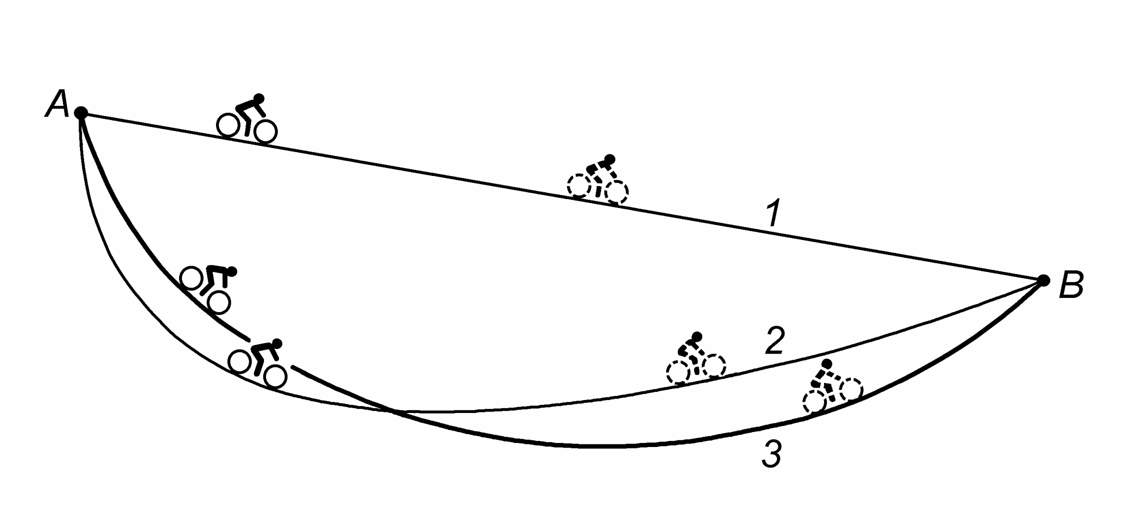

FIGURE 1. Cycloid is the curve of the quickest moving down.

If you want to make sure of brachistochrone and tautochrone properties of the cycloid it is necessary to plot this curve. To do that cut a 80 mm circle from the piece of cardboard and put a mark on the edge of it. Put a ruler on the sheet of thick paper and move the circle close to it until the mark touches the ruler (Fig. 2); for good layout a radius is drawn across the circle, the end of the radius being the mark. With a pencil mark a dot on the paper near the mark and roll the circle along the ruler without slipping marking consecutive positions of the mark on the sheet of paper. Continue until the circle rolls over the full turn, then connect the marked dots with a smooth curve and you will get the arc of cycloid. Carefully cut the paper along the plotted line — the obtained template will be helpful for subsequent constructing cycloidal guides.

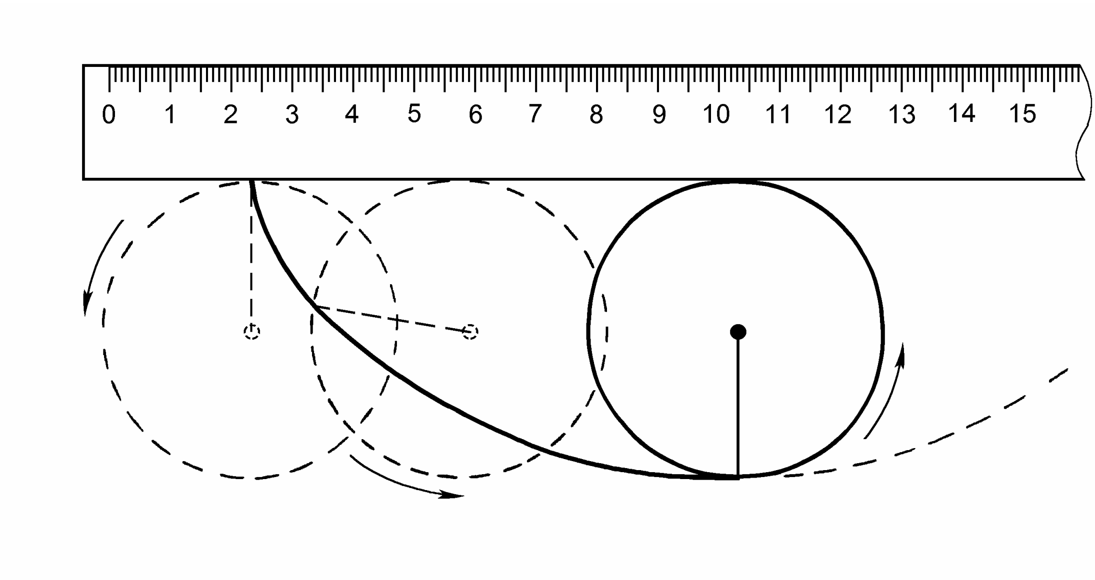

FIGURE 2. How to plot the cycloid.

The basic difficulty of the actual observation of brachistochrone and tautochrone properties of the cycloid is that the time of motion along the cycloidal chute is too short to visually notice the contemporaneity or lag of one of the balls. It is possible to increase the time of the motion of balls without increasing the length of the trajectory by ‘decreasing the acceleration of free fall’. To do that it is sufficient to turn the cycloid plane around the horizontal axis so that the angle between it and horizontal would be small. In this case the essence of the phenomenon won't change, but the projection of **g** on the cycloid plane will play the role of the acceleration of the free fall.

Figure 3 illustrates an outline of the device which makes possible to see that the cycloid possesses brachistochrone and tautochrone properties. A rectangular duralumin or plastic plate 1, dimensions 2 x 200 x 280 mm, is set up on the surface of the table by means of a block 6 on the angle of 5 to 15 degrees. Plate 2 is attached on it through gaskets 5 by means of screws 4, the upper edge of the former being the cycloid arc carved with a fretsaw according to the above described template. A cycloid plate 3 is fixed to plate 2, the upper edge of the former is identical to that of plate 2, i.e. it is shaped as an arc of the cycloid same dimensions. The upper edges of plates 2 and 3 are situated just one under the other. Plate 7 is set up in the gap between plates 1 and 2 so that it can be turned around screw 4 in the upper left-hand corner of the device, the shape of the upper edge of the former being a random curve or just a straight line. A part of the guiding plate 7 projecting outside dimensions of the device is used as a handle for proper positioning of the plate. Two identical 7 to 9 mm steel balls are required for the experiment. The use of thicker plates, balls of a greater diameter and proportional change of the device dimensions are acceptable.

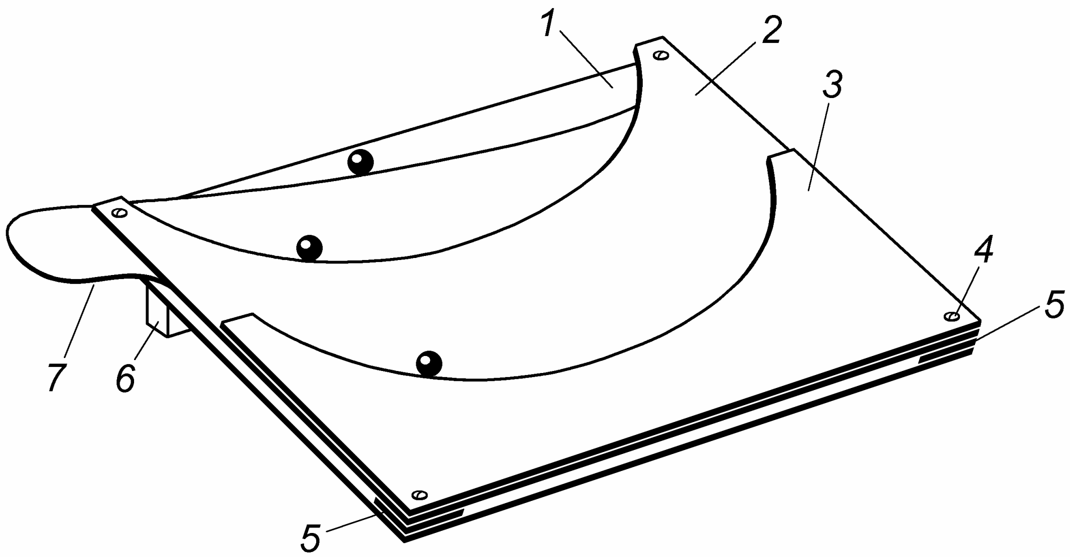

FIGURE 3. Device for demonstrating mechanical properties of cycloid.

The balls are set on non-cycloid and cycloid guides (upper edges of plates 7 and 2) in the upper left-hand corner of the device and let them free simultaneously. The second ball takes over the first one which is moving along non-cycloid trajectory and reaches the lowest point of two guides intersection earlier where it falls onto plate 1. After the balls have stopped the sequence of their position is seen on the right-hand (see Figure 3) part of the cycloid trajectory coinciding with the sequence of their arrival at the said point. If non-cycloid guide is turned or substituted with another one the result of the experiment will not change.

Tautochrone property of the cycloid can be observed in the following way. Place the balls at any points of upper edges of the cycloidal guides and let them free simul-taneously. You would see that the balls rolling down along the guides irrespectively of start points reach the lowest points of the cycloidal guides simultaneously and then having passed them inertially they stop at the upper points of their trajectories simultaneously, return, etc. That is to say, the balls oscillate in phase with different amplitudes but equal periods which proves the tautochroneity of the cycloid.

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