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Solid State Communications 140 (2006) 236–239
solid state communications
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Influence of excited Landau levels on a two-dimensional electron–hole system in a strong perpendicular magnetic field

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Received 9 May 2006; received in revised form 3 August 2006; accepted 7 August 2006 by L.V. Keldysh
Available online 24 August 2006

Abstract

The study of the quantum states of a two-dimensional electron–hole system in a strong perpendicular magnetic field is carried out with special attention to the influence of virtual quantum transitions of interacting particles between the Landau levels. These virtual quantum transitions from the lowest Landau levels to excited Landau levels with arbitrary quantum numbers n and m and their reversion to the lowest Landau levels in second order perturbation theory result in an indirect attraction between the particles. The influence of the indirect interaction on the magnetoexciton ground state, on the chemical potential of the Bose–Einstein condensed magnetoexcitons, and on the ground state energy of the metallic-type electron–hole liquid is investigated in the Hartree–Fock approximation. The coexistence of different phases is suggested.

PACS: 71.35.Ji; 71.35.Lk; 71.35.Ee

Keywords: A. Magnetoexcitons; D. Bose–Einstein condensation

Properties of atoms and excitons are dramatically changed in strong magnetic fields, such that the distance between Landau levels $\hbar\omega_c$ exceeds the corresponding Rydberg energies and the magnetic length $l = \sqrt{\hbar c/eH}$ is small compared to their Bohr radii [1,2]. Even more interesting phenomena are exhibited in the case of two-dimensional (2D) electron systems due to the quenching of kinetic energy in a high magnetic field, with the representative example being integer and fractional Quantum Hall effects [3,4]. Properties of the 2D electron–hole (e–h) system in a strong magnetic field attracted great attention during the past two decades [5–12]. Collective states such as the Bose–Einstein condensation (BEC) of magnetoexcitons and the formation of a metallic-type electron–hole liquid (EHL) were investigated in Refs. [5–11].

The purpose of the present paper is the detailed study of the influence of virtual quantum transitions of Coulomb interacting particles from the LLLs to all excited Landau levels (ELLs). We consider three aspects of the problem: the influence on the chemical potential of the BEC-ed magnetoexcitons, on the energy per pair in the composition of EHL and electron–hole droplets (EHD), as well as the influence on the wave function and on the energy level of a single magnetoexciton. The first steps in this direction were made in Ref. [11].

The full Hamiltonian consists of a zero order Hamiltonian H_0 describing the Landau quantization of free quasiparticles and the Hamiltonian H_{Coul} of Coulomb interaction:

$$H = H_0 + H_{\mathrm{Coul}}, \tag{1}$$

where

$$\begin{aligned}\hat{H}_{\mathrm{Coul}} = \frac{1}{2}\sum_{p,q,s}\sum_{n,m,n',m'} [&F_{e-e}(p,n;q,m;p-s,n';q+s,m')\\ &\times a^{\dagger}_{n,p}a^{\dagger}_{m,q}a_{m',q+s}a_{n',p-s}\\ &+ F_{h-h}(p,n;q,m;p-s,n';q+s,m')\\ &\times b^{\dagger}_{n,p}b^{\dagger}_{m,q}b_{m',q+s}b_{n',p-s}]\\ &- \sum_{p,q,s}\sum_{n,m,n',m'} F_{e-h}(p,n;q,m;p-s,n';q+s,m')\\ &\times a^{\dagger}_{n,p}b^{\dagger}_{m,q}b_{m',q+s}a_{n',p-s}.\end{aligned} \tag{2}$$

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0038-1098/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ssc.2006.08.010

The operators $a^{\dagger}_{n,p}$ and $a_{n,p}$ stand for creation and annihilation of an electron in the state with Landau level (LL) index n and the momentum p; $b^{\dagger}_{n,q}$ and $b_{n,q}$ hold the same meaning for holes. The number of discrete states is $N = S/2\pi l^2$, where S is the layer area.

The matrix elements of the two-particle Coulomb interaction are given by

$$F_{i-j}(p,n;q,m;p-s,n';q+s,m') = \iint \mathrm{d}\boldsymbol{\varrho}_1 \mathrm{d}\boldsymbol{\varrho}_2 \psi^{i*}_{n,p}(\boldsymbol{\varrho}_1)\psi^{j*}_{m,q}(\boldsymbol{\varrho}_2)\frac{e^2}{\epsilon_0|\boldsymbol{\varrho}_1-\boldsymbol{\varrho}_2|} \times \psi^{i}_{n',p-s}(\boldsymbol{\varrho}_1)\psi^{j}_{m',q+s}(\boldsymbol{\varrho}_2), \quad i,j = e,h. \tag{3}$$

Here $\psi^{e,h}_{n,p}(\boldsymbol{\varrho})$ is a mixed basis set of Landau functions and plane waves [13]. In the following we separate from the total Coulomb interaction term the interaction within the LLLs denoted as $H^{\mathrm{LLL}}_{\mathrm{Coul}}$ and the terms of the type

$$F_{i-j}(0,p;0,q;n,p-s;m,q+s), \quad F_{j-i}(n,p;m,q;0,p-s;0,q+s) \tag{4}$$

which describe the virtual quantum transitions from the initial states $n = m = 0$ to ELLs $n, m \neq 0$ and their reversion to the LLLs. A recurrence relation for all ELLs was derived calculating matrix elements with quantum numbers $n = 0, 1, 2, 3, 4$. The part of the Coulomb interaction containing the matrix elements of Eq. (4) is denoted $H^{\mathrm{ELL}}_{\mathrm{Coul}}$. All other terms entering Eq. (2) were neglected.

On the next step the terms of the type Eq. (4) were excluded from the Hamiltonian equation (2) by means of the unitary transformation $\hat{U} = e^{i\hat{S}}$ [14], where $\hat{S} = \hat{S}^{\dagger}$ and is determined from the equation

$$i[\hat{H}_0, \hat{S}] + H^{\mathrm{ELL}}_{\mathrm{Coul}} = 0. \tag{5}$$

The new transformed Hamiltonian H_{eff} is determined as

$$H_{\mathrm{eff}} = {}_{\mathrm{ELL}}\langle 0|\, e^{-iS}\hat{H}e^{iS}|0\rangle_{\mathrm{ELL}} \simeq H^{\mathrm{LLL}}_{\mathrm{Coul}} + \frac{i}{2}\,{}_{\mathrm{ELL}}\langle 0|[H^{\mathrm{ELL}}_{\mathrm{Coul}}, \hat{S}]|0\rangle_{\mathrm{ELL}}. \tag{6}$$

Here the average is made using the vacuum state for ELLs $|0\rangle_{\mathrm{ELL}}$.

Introducing the chemical potential of electrons μ_e and of holes μ_h, the effective Hamiltonian H_{eff} takes the final form:

$$H_{\mathrm{eff}} = -\mu_e\sum_p a^{\dagger}_p a_p - \mu_h\sum_p b^{\dagger}_p b_p + H^{\mathrm{LLL}}_{\mathrm{Coul}} - \frac{1}{2}\sum_{p,q,z}\phi_{e-e}(p,q,z)a^{\dagger}_p a^{\dagger}_q a_{q+z}a_{p-z} - \frac{1}{2}\sum_{p,q,z}\phi_{h-h}(p,q,z)b^{\dagger}_p b^{\dagger}_q b_{q+z}b_{p-z} - \sum_{p,q,z}\phi_{e-h}(p,q,z)a^{\dagger}_p b^{\dagger}_q b_{q+z}a_{p-z}. \tag{7}$$

Here the matrix elements $\phi_{i-j}(p,q,z)$ of indirect interaction are defined as follows

$$\phi_{i-j}(p,q,z) = \sum_{n,m}\frac{\phi_{i-j}(p,q,z;n,m)}{n\hbar\omega_{ci} + m\hbar\omega_{cj}},$$

$$\phi_{i-j}(p,q,z;n,m) = \sum_t F_{i-j}(p,0;q,0;p-t,n;q+t,m) \times F_{i-j}(p-t,n;q+t,m;p-z,0;q+z,0). \tag{8}$$

Now we will discuss the influence of the supplementary indirect interaction on the ground state energies of two collective phases formed by a 2D e–h system in a strong perpendicular magnetic field. One of them is a metallic type EHL and another is the BEC of magnetoexcitons on the single-particle state with wave vector $\mathbf{k}$. Both of them will be discussed below in the Hartree–Fock approximation (HFA). Considering the EHL we start with the effective Hamiltonian equation (7) but without chemical potential μ_e and μ_h. We calculate the ground state energy at $T = 0$, when the average values of the electron and hole numbers on the LLLs are equal to their filling factor ν^2

$$\left\langle a^{\dagger}_p a_p\right\rangle = \left\langle b^{\dagger}_p b_p\right\rangle = \nu^2. \tag{9}$$

Applying Wick's theorem we obtained the ground state energy in HFA

$$E_{\mathrm{EHL}} = -N_{e-h}[\nu^2 I_l + \nu^2(2A - B)], \tag{10}$$

with $N_{e-h} = N\nu^2$. The coefficients A and B are determined in the case of electrons and holes with equal masses $m_e = m_h$ and cyclotron frequencies $\omega_{ce} = \omega_{ch} = \omega_c$. In this case one can write the sum of the diagonal matrix elements of the indirect interaction as

$$A_{i-j} = \frac{I_l^2}{\pi\hbar\omega_c}\sum_{n\geqslant 1}\sum_{m\geqslant 1}\frac{(n+m-1)!}{2^{n+m}n!m!(n+m)} = \frac{I_l^2}{\pi\hbar\omega_c}S, \tag{11}$$

where $S \approx 0.481$. Here $I_l = \sqrt{\pi/2}\,e^2/\epsilon_0 l$ is the magnetoexciton binding energy, where ϵ_0 is the background dielectric constant. In a similar way we have found the sum of the nondiagonal matrix elements, which determines the exchange contribution of the indirect supplementary interaction:

$$B_{i-i} = \frac{2I_l^2}{\pi\hbar\omega_c}\sum_{n,m}\frac{1}{2^{n+m}n!m!(n+m)}\int_0^{\infty}\mathrm{d}x e^{-\frac{x^2}{2}}x^{n+m} \times \int_0^{\infty}\mathrm{d}y e^{-\frac{y^2}{2}}y^{n+m}J_{n+m}(xy) = \frac{2I_l^2}{\pi\hbar\omega_c}T, \tag{12}$$

where $J_n(z)$ is Bessel function of the first kind. After a thorough analysis of the sum in the above expression it can be shown that $T \approx 0.2161$. The energy per one e–h pair E_{EHL} of EHL in units of I_l is then given by

$$\frac{E_{\mathrm{EHL}}}{I_l} = -\nu^2\left[1 + \frac{2I_l}{\pi\hbar\omega_c}(S-T)\right] = -\nu^2\left(1 + 0.168\frac{I_l}{\hbar\omega_c}\right) \tag{13}$$

The lowest energy is achieved at a filling factor $v^2 = 1$ and it determines the energy per pair inside the EHD to be equal to

$$\frac{E_{\rm EHD}}{I_l} = -(1 + 0.168r); \quad r = \frac{I_l}{\hbar\omega_c}. \tag{14}$$

The ratio r must be less than 1 to satisfy the condition of a strong magnetic field.

Consider the BEC of magnetoexcitons in a single particle state with wave vector $k \neq 0$ in the Hartree–Fock–Bogoliubov approximation (HFBA). As was demonstrated in the papers [9, 10] the BEC can be introduced into the starting Hamiltonian equation (7) by means of the canonical transformation

$$\begin{aligned} a_p &= u\alpha_p + v\left(p - \frac{k_x}{2}\right)\beta^\dagger_{k_x-p}; \\ b_p &= u\beta_p - v\left(\frac{k_x}{2} - p\right)\alpha^\dagger_{k_x-p}, \end{aligned} \tag{15}$$

with the coefficients

$$v(t) = v\exp[-{\rm i}k_y t l^2]; \quad v^2 + u^2 = 1.$$

The normal ordering in the operators $\alpha^\dagger_p, \alpha_p, \beta^\dagger_p, \beta_p$ of the transformed Hamiltonian will generate in the HFBA the quadratic term H_2, similar to the quadratic Hamiltonian Equation (39) of Ref. [10], which contains the coefficients $E(\mathbf{k}, v^2, \mu)$ and $\psi(\mathbf{k}, v^2, \mu)$ defined by the formulas Eqs. (40) and (41) of Ref. [10]. The quadratic Hamiltonian H_2 is given below for the case of electrons and holes with equal masses $m_e = m_h$, cyclotron frequencies $\omega_{ce} = \omega_{ch} = \omega_c$ and chemical potentials $\mu_e = \mu_h = \mu/2$:

$$\begin{aligned} H_2 = &\sum_p [E(\mathbf{k}, v^2, \mu) + (B - 2A)v^2(1 - 2v^2) \\ &+ 2v^2(1 - v^2)\Delta(k)] \times (\alpha^\dagger_p\alpha_p + \beta^\dagger_p\beta_p) \\ &+ \sum_p \left[uv\left(\frac{k_x}{2} - p\right)\beta_{k_x-p}\alpha_p + uv\left(p - \frac{k_x}{2}\right)\alpha^\dagger_p\beta^\dagger_{k_x-p}\right] \\ &\times\{-\psi(\mathbf{k}, v^2, \mu) + 2v^2(B - 2A + \Delta(k)) - \Delta(k)\}. \end{aligned} \tag{16}$$

Here $\Delta(k)$ is determined by the sum

$$\begin{aligned} \Delta(k) = &\frac{I_l^2\exp(-k^2l^2)}{\pi}\sum_{n,m}\frac{(kl)^{2|n-m|}}{n!m!2^{|n-m|}(n\hbar\omega_{ce} + m\hbar\omega_{ch})} \\ &\times\left[\frac{\Gamma\left(\frac{m+n+|n-m|+1}{2}\right)}{\Gamma(|n-m|+1)}\,{}_1F_1\left(\frac{|n-m|+1-(n+m)}{2};\right.\right. \\ &\left.\left.|n-m|+1;\frac{(kl)^2}{2}\right)\right]^2. \end{aligned} \tag{17}$$

At the point $k = 0$ and $\omega_{ce} = \omega_{ch} = \omega_c$ we have

$$\Delta(0) = \frac{2I_l^2}{\pi\hbar\omega_c}(\pi\ln 2 - 2G) = \frac{2I_l^2}{\pi\hbar\omega_c}0.344, \tag{18}$$

where G is Catalan's constant ($\simeq$0.915966).

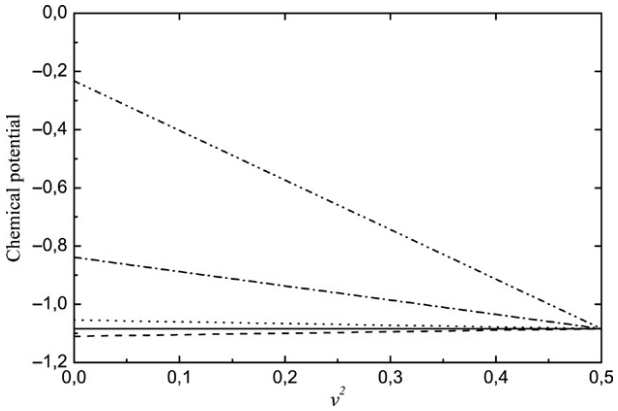

Fig. 1. Chemical potential in units of exciton binding energy I_l versus filling factor v^2 of the LLL for $r = 1/2$. Solid line: energy per e–h pair in EHD phase; dashed line: chemical potential of condensed excitons with $k = 0$; dotted line: the same, but for $kl = 0.5$; dash-dotted line: the same, but for $kl = 1$; dash-dot-dot line: the same, but for $kl = 3.6$.

Following the notations of Eqs. (40) and (41) of Ref. [10] we have

$$\begin{aligned} E(\mathbf{k}, v^2, \mu) &= 2v^2u^2I_{ex}(k) + I_l(v^4 - v^2u^2) - \frac{\mu}{2}(u^2 - v^2), \\ \psi(\mathbf{k}, v^2, \mu) &= 2v^2I_l + I_{ex}(k)(1 - 2v^2) + \mu. \end{aligned} \tag{19}$$

Requiring the last bracket in Eq. (16) to be equal to zero, i.e. compensating the dangerous diagrams describing the spontaneous creation and annihilation of e–h pairs in the vacuum state, one can obtain the chemical potential μ of the system in the HFBA:

$$\begin{aligned} \mu^{\rm HFB} &= -\tilde{I}_{ex}(k) + 2v^2(B - 2A + \tilde{I}_{ex}(k) - I_l) \\ &= -\tilde{I}_{ex}(k) + 2v^2(B - 2A + \Delta(k) - E(k)). \end{aligned} \tag{20}$$

Here the renormalized ionization potential of magnetoexcitons $\tilde{I}_{ex}(k)$ containing the correction due to influence of all ELLs was introduced:

$$\begin{aligned} &\tilde{I}_{ex}(k) = I_{ex}(k) + \Delta(k); \\ &I_{ex}(k) = I_l\exp[-k^2l^2/4]I_0(k^2l^2/4); \;\; E(k) = I_l - I_{ex}(k), \end{aligned} \tag{21}$$

where $I_0(z)$ is the modified Bessel function of the first kind. The dependence of the chemical potential $\mu^{\rm HFB}$ in HFBA and the energy per e–h pair in the composition of EHD versus the filling factor v^2 of the LLL for different values of the dimensionless wave vector $kl = 0, 0.5, 1.0, 3.6$ and for $r = 0.5$ is presented in Fig. 1. At small values of $kl < 0.5$ the plotted curve $\mu(k, v^2)$ has a positive slope, corresponding to the stability of the ground state in the HFBA. The supplementary indirect e–e and h–h interactions being averaged in the HFA gives rise to direct and exchange pairing terms. The former terms, being negative, increase the binding energy of magnetoexcitons and energy per pair in the EHL phase, whereas the later ones are repulsive. They diminish the influence of the direct pairing terms, but do not surpass them, so that the resulting influence of both terms remains attractive. In the BEC phase the supplementary e–h attraction after the Bogoliubov's u-v transformation equation (15) yields a repulsive-type Bogoliubov self-energy term and

results in the stability of the BEC phase of magnetoexcitons with small momenta. Note, however, the energy per e–h pair in EHD is close on the energy scale to the values of chemical potential μ^{HFB}. This means that these states can coexist being realized in different areas of the sample. From other arguments the coexistence of a degenerate Bose–gas with small momenta and of drops of metastable dielectric liquid phase formed by Bose–Einstein condensed magnetoexcitons with considerable values of wave vectors $kl > 3$ was suggested in Ref. [15]. Taking into account the results of the present paper we arrived at the conclusion that the metallic-type electron–hole droplet can coexist with both dielectric gaseous and liquid phases of condensed magnetoexcitons.

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