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Collective Elementary Excitations of Two-Dimensional Magnetoexcitons in the Bose-Einstein Condensation State

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The collective elementary excitations of a system of two-dimensional magnetoexcitons in a state of Bose-Einstein condensation (BEC) with arbitrary wave vector was investigated in Hartree-Fock-Bogoliubov approximation. The breaking of the gauge symmetry of the Hamiltonian was introduced following the idea proposed by Bogoliubov in his theory of quasi-averages. The equations of motion were written in the frame of the starting electron and hole creation and annihilation operators. The chains of equations of motion for a set of Green's functions describing the exciton-type excitations as well as the plasmon-type excitations were deduced. Their disconnections were introduced using the perturbation theory with a small parameter of the theory proportional to the filling factor multiplied by the phase space filling factor. The energy spectrum of the collective elementary excitations is characterized by the interconnection of the exciton and plasmon branches, because the plasmon-type elementary excitations are gapless and are lying in the same spectral interval as the exciton-type elementary excitations.

Keywords: Bose-Einstein Condensation, Magnetoexciton, Collective Properties.

CONTENTS

1. INTRODUCTION

Properties of atoms and excitons are dramatically changed in strong magnetic fields, such that the distance between Landau levels $\hbar\omega_c$, exceeds the corresponding Rydberg energies R_y and the magnetic length $l = \sqrt{\hbar c/eH}$ is small compared to their Bohr radii.[1,2] Even more interesting phenomena are exhibited in the case of two-dimensional (2D) electron systems due to the quenching of the kinetic energy at high magnetic fields, with the representative example being integer and fractional Quantum Hall effects.[3–5] The discovery of the FQHE[6–8] changed fundamentally the established concepts about charged elementary excitations in solids.[5] The notion of the incompressible quantum liquid (IQL) was introduced in Ref. [7] as a homogeneous phase with the quantized densities $v = p/q$, where p is an integer and $q \neq 1$ is odd having charged elementary excitations with a fractional charge $e^* = \pm e/q$. These quasiparticles were named as anyons. A classification for free anyons and their hierarchy were studied in Refs. [9, 10]. An alternative concept to hierarchical scheme was proposed in Ref. [11], where the notion of composite fermions (CF) was introduced. The CF consists from the electron bound to an even number of flux quanta. In the frame of this concept the FQHE of electrons can be physically understood as a manifestation of the IQHE of CFs.[11] The statistics of anyons was determined in Refs. [10, 12]. It was established that the wave function of the system changes by a complex phase factor $\exp[i\pi\alpha]$, when the quasiparticles are interchanged. For bosons $\alpha = 0$, for fermions $\alpha = 1$ and for anyons with $e^* = -e/3$ their statistical charge is $\alpha = -1/3$. As was shown in Ref. [13], there were no soft branches of neutral excitations in IQL. The energy gap

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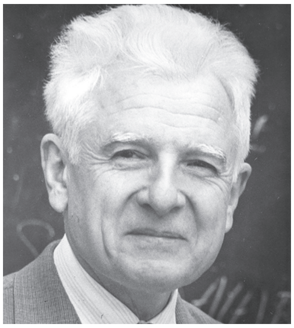
Δ for formation of a quasielectron–quasihole pair has the scale of Coulomb energy $E_Q = e^2/\varepsilon l$, where ε is the dielectric constant of the background. However delta was found to be small $\Delta \simeq 0.1E_Q$. The lowest branch was called as magnetoroton[13] and can be modelled as a quasiexciton.[5] As was mentioned in Ref. [5] the traditional methods and concepts based either on the neglecting of the electron–electron interaction or on self-consistent approximation are inapplicable to IQL. In a strong magnetic field the binding energy of an exciton increases from R_y to I_l.

There are two another small parameters of the theory. One of them determines how strong the magnetic field strength H is, and it verifies whether the starting supposition of a strong magnetic field is fulfilled. This parameter is expressed by the ratio $I_l/(\hbar\omega_c) < 1$. Here I_l is the magnetoexciton ionization potential, ω_c is the cyclotron



Moskalenko Sveatoslav Anatol was born on September 26, 1928 in the village Bravicha Calarash district, Republic of Moldova. He graduated the Kishinev State University in 1951. In 1956–1959 he was post-graduate student of the Institute of Physics of the Academy of Sciences of Ukrainian SSR. He became Candidate of Physico-mathematical Sciences equivalent to PHD degree in 1961. Doctor of Physico-mathematical Sciences and the professor in the field of theoretical and mathematical physics beginning with 1971 and 1974 correspondingly. He became Laureate of the State prize in the field of Sciences and Technics of MSSR and USSR in 1981 and 1988 correspondingly. He was elected member-correspondent and full member of the Academy of Sciences of Moldova in 1989 and 1992 correspondingly. The results concerning the Bose-Einstein condensation of excitons and biexcitons as well the coherent nonlinear optics with excitons were reviewed in the monograph written together with Prof. D. W. Snoke from Pittsburg University. Due to the collaboration with Prof. M. A. Liberman from Uppsala University last years the properties of excitons in a strong magnetic field are studied. The polarizability, correlation energy and the dielectric liquid phase of Bose-Einstein condensed 2D magnetoexcitons with motional dipole moments were studied. The possible existence of the metastable dielectric liquid phase formed by Bose-Einstein condensed magnetoexcitons with wave vectors and motional dipole moments different from zero was established theoretically.

Michael (Mikhail) Liberman was born in Moscow, USSR on October 23, 1942. He graduated from Moscow State University in 1966. From 1969 to 2003 he worked at P. Kapitsa Institute for Physical Problems, Academy of Sciences USSR. He received his Ph.D. in 1971 from P. Lebedev Physical Institute in Moscow for the group theory in quantum mechanics and invariant expansion of the relativistic amplitudes, and then his Doctor of Physical and Mathematical Sciences degree in 1981 for a thesis on ionizing shock waves. Since 1991, he is professor of theoretical statistical physics working at the Physics Department, Uppsala University, Sweden. He is a citizen of both Russia and the Sweden. Among his achievements are the nonlinear theory of electromagnetic wave propagating in nonequilibrium plasmas (for example, in the ionosphere); a theory of the ionizing shock waves, exact solution for a hydrogen atom in a magnetic field of arbitrary strength, theory of a hydrogen molecule in a strong magnetic field, non-stationary nonlinear equation for a curved flame, theory of type Ia supernova explosion. At Uppsala University he continue his work on combustion theory, for which he was recently nominated for Gold Medal of Combustion Institute, and he is also focused on the research of the Bose-Einstein condensate of excitons in low dimensional semiconductors in a strong magnetic field. He is author of the books: Physics of Shock Waves in Gases and Plasmas, Springer-Verlag, 1985 (with A. Velikovich), Physics of High-Density Z-pinch Plasmas, Springer-Verlag, 1998 (with J. DeGroot, A. Toor, R. Spielman), Introduction to Physics and Chemistry of Combustion, Springer-Verlag, 2008.

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frequency $eH/\mu c$ calculated with the reduced mass μ and the magnetic length l equals to $\sqrt{\hbar c/eH}$. Another small parameter has a completely different origin and is related with the concentration of the electron–holes (e–h) pairs. In our case it can be expressed as a product of the filling factor $v = \mathrm{v}^2$ and of another factor $(1-\mathrm{v}^2)$ which reflects the Pauli exclusion principle and the phase-space filing (PSF) effect. This compound parameter $\mathrm{v}^2(1-\mathrm{v}^2)$ in the case of Bose-Einstein condensed excitons can take the form $u^2\mathrm{v}^2$, where u, v are Bogoliubov transformation coefficients and $u^2 = (1-\mathrm{v}^2)$. The both small parameters will be used below. But in the case of FQHE the filling factor $v = \mathrm{v}^2$ basically determines the underlying physics and it can not be changed arbitrarily. Instead of the perturbation theory on the filling factor v the exact numerical diagonalization for a few number of particles $N \leq 10$ proved to be the most powerful tool in studies of such systems.[5] The spherical geometry for these calculations was proposed,[10,14] considering a few number of particles on the surface of a sphere with the radius $R = \sqrt{S}l$, whereas S is the dimensional Haldane parameter, so as the density of the particles on the sphere to be equal with the filling factor of 2DEG. The magnetic monopole in the center of the sphere creates a magnetic flux through the sphere $2S\phi_0$, which is multiple to the flux quantum $\phi_0 = 2\pi\hbar c/e$. The angular momentum L of a quantum state on the sphere and the quasimomentum k of the FQHE state on the plane obey the relation $L = Rk$. Spherical model is characterized by continuous rotational group, which is analogous with the continuous translational symmetry in the plane.

Properties of the symmetric 2D electron–hole (e–h) system, with equal concentrations for both components, with coincident matrix elements of Coulomb electron–electron, hole–hole and electron–hole interactions in a strong perpendicular magnetic field also attracted a great attention during last two decades.[15–22] A hidden symmetry and the multiplicative states were discussed in many papers.[19,23,24] The collective states such as the Bose-Einstein condensation (BEC) of two-dimensional magnetoexcitons and the formation of metallic-type electron–hole liquid (EHL) were investigated in Refs. [15–22]. The search for Bose-Einstein condensates has became a milestone in the condensed matter physics.[25] The remarkable properties of super fluids and superconductors are intimately related to the existence of a bosonic condensate of composite particles consisting of an even number of fermions. In highly excited semiconductors the role of such composite bosons is taken on by excitons, which are bound states of electrons and holes. Furthermore, the excitonic system has been viewed as a keystone system for exploration of the BEC phenomena, since it allows to control particle densities and interactions *in situ*. Promising candidates for experimental realization of such system are semiconductor quantum wells (QWs),[26] which have a number of advantages compared to the bulk systems. The coherent pairing of electrons and holes occupying only the lowest Landau levels (LLLs) was studied using the Keldysh-Kozlov-Kopaev method and the generalized random-phase approximation.[20,27] The BEC of magnetoexcitons takes place in a single exciton state with wave vector k, supposing that the high density of electrons in the conduction band and of holes in the valence band were created in a single QW structure with size quantization much greater than the Landau quantization. In the case $k \neq 0$ a new metastable dielectric liquid phase formed by Bose-Einstein condensed magnetoexcitons was revealed.[20,21] The importance of the excited Landau levels (ELLs) and their influence on the ground states of the systems was first noticed by the authors of the papers.[16–19] The influence of the excited Landau levels (ELLs) of electrons and holes was discussed in details in paper.[21,22] The indirect attraction between electrons (e–e), between holes (h–h) and between electrons and holes (e–h) due to the virtual simultaneous quantum transitions of the interacting charges from LLLs to the ELLs is a result of their Coulomb scattering. The first step of the scattering and the return back to the initial states were described in the second order of the perturbation theory.

Now the short review of the intra-LLLs excitations in the 2D two-component electron–electron and electron–hole (e–h) gases will be presented.

Das Sarma and Madhukar[28] have investigated theoretically the longitudinal collective modes of spatially separated two-component two-dimensional plasma in solids using the generalized random phase approximation. It can be realized in semiconductors heterojunctions and superlattices. The two-layer structure with two-component plasma is discussed below. It has long been known that a two-component plasma has two branches to it longitudinal oscillations. The higher frequency branch is named as optical plasmon (OP). Here the two carries densities of the same signs oscillate in-phase and their density fluctuation operators $\hat{\rho}_{e,1}(\vec{Q})$ and $\hat{\rho}_{e,2}(\vec{Q})$ form an in-phase superposition

$$\hat{\rho}_{\mathrm{OP}}(\vec{Q}) = \hat{\rho}_{e,1}(\vec{Q}) + \hat{\rho}_{e,2}(\vec{Q})$$

In the case of opposite signs electron and hole charges they oscillate out-of-phase and their charge density fluctuation operators $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$ combine in out-of-phase manner

$$\hat{\rho}_{\mathrm{OP}}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q})$$

The lower frequency branch is named as acoustical plasmon (AP). Now the carriers of different signs oscillate in-phase, whereas the carriers of the same signs oscillate out-of-phase. Their charge density fluctuation operators combine in the form

$$\hat{\rho}_{\mathrm{AP}}(\vec{Q}) = \hat{\rho}_{e,1}(\vec{Q}) - \hat{\rho}_{e,2}(\vec{Q}); \quad \hat{\rho}_{AP}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q})$$

The optical and acoustical branches of two-component electron plasma have the dispersion relations in the long wavelength region as follows

$$\omega_{\mathrm{OP}}(q) \sim \sqrt{q}; \quad \omega_{\mathrm{AP}}(q) \sim q; \quad q \to 0$$

By virtue of spatial separation z between the two components of the 2D plasma the AP branch becomes with a greater slope of the linear q dependence, because this slope is proportional to z, when z is of the order of Bohr radius a_B. At small $z \to 0$ the AP branch lies inside the single-particle excitation spectrum of the faster moving charged carriers. They leave the corresponding Fermi seas crossing the Fermi energies of the degenerate Fermi gases. This single-particle spectrum is severely Landau damped. At larges values $z > a_B$ the Coulomb interaction between charges in different layers can be neglected and each layer supports an ordinary 2D plasma oscillations with a dispersion $\omega_P(q) \sim q^{1/2}$.[28]

The plasmon oscillations in one-component system on the monolayer in a strong perpendicular magnetic field were studied by Girvin, MacDonald and Platzman,[13] who proposed the magnetoroton theory of collective excitations in the conditions of the fractional quantum Hall effect (FQHE). The FQHE occurs in low-disorder, high-mobility samples with partially filled Landau levels with filling factor of the form $\nu = 1/m$, where m is an integer, for which there is no single-particle gap. Considerable progress has recently been achieved toward understanding the nature of the many-body ground state well described by Laughlin variational wave function.[7] The theory of the collective excitation spectrum proposed by Ref. [13] is closely analogous to Feynman's theory of superfluid helium.[29] The main Feynman's arguments lead to the conclusions that on general grounds the low lying excitations of any system will include density waves. As regards the 2D system the perpendicular magnetic field quenches the single particle continuum of kinetic energy leaving a series of discrete highly degenerate Landau levels spaced in energy at intervals $\hbar\omega_c$. In the case of filled Landau level $\nu = 1$ because of Pauli exclusion principle the lowest excitation is necessarily the cyclotron mode in which particles are excited into the next Landau level. In the case of FQHE the lowest Landau level (LLL) is fractionally filled. The Pauli principle no longer excludes low-energy intra-Landau-level excitations. For the FQHE case the primary importance have the low-lying excitations, rather than the high-energy inter-Landau-level cyclotron modes.[13] The spectrum has a relatively large excitation gap at zero wave vector $kl = 0$ and in addition it exhibits a deep magneto-roton minimum at $kl \sim 1$ quite analogous to the roton minimum in helium. The magneto-roton minimum becomes more deeper and deeper at the decreasing of the filling factor ν in the row $1/3, 1/5, 1/7$ and is the precursor to the gap collapse associated with the Wigner crystallization which occurs at $\nu = 1/7$. For largest wave vectors the low lying mode crosses over from being a density wave to becoming a quasiparticle excitation.[13] The Wigner crystal transition occurs slightly before the roton mode goes completely soft. The magnitude of the primitive reciprocal lattice vector for the crystal lies close to the position of the magneto-roton minimum. The autors of Ref. [13] suggested also the possibility of pairing of two rotons of opposite momenta leading to the bound two-roton state with small total momentum, as it is known to occur in helium. In difference from the case of fractional filling factor, the excitations from a filled Landau level in the 2DEG were studied by Kallin and Halperin.[30]

Fertig[31] investigated the excitation spectrum of two-layer and three-layer electron systems. In particular case the two-layer system in a strong perpendicular magnetic field with filling factor $\nu = 1/2$ of the lowest Landau level (LLL) in the conduction band of each layer was considered. Inter-layer separation z was introduced. The spontaneous coherence of two-component two-dimensional (2D) electron gas was introduced.

Both half filled layers a and b are accompanied by a substrate with positive charge guaranteeing the electrical neutrality of the system. The half filled layer a can be considered as a full filled with electrons in the LLL of the conduction band and a half filled by holes in the LLL of the same conduction band.

The electrons of the full filled conduction band are compensated by the charge of the substrate and we can only consider the electrons on the layer b and the holes on the layer a.

Then the wave function[31] of the coherent two-layer electron system can be rewritten in the form which coincides with the BCS-type wave function of the superconductor. It represents the coherent pairing of the conduction electrons on the LLL of the layer b with the holes in the LLL of the conduction band of the layer a and describes the BEC of such unusual excitons named as FQHE excitons, because they appear in the conditions proper to the observation of the fractional quantum Hall effect. Here only the BEC on the single exciton state with wave vector $\vec{k} = 0$ is considered.

Fertig has determined the energy spectrum of the elementary excitations in the frame of this ground state. In the case of $z = 0$ the lowest-lying excitations of the system are the higher energy excitons.

Because of the neutral nature of the $\vec{k} = 0$ excitons the dispersion relation of these excitations is given in a good approximation by $\hbar\omega(k) = E_{\mathrm{ex}}(k) - E_{\mathrm{ex}}(0)$, where $E_{\mathrm{ex}}(k)$ is the energy of exciton with wave vector $\vec{k}$. This result was first obtained by Paquet, Rice and Ueda[19] using a random phase approximation (RPA). In the case $z = 0$ the dispersion relation $\omega(k)$ vanishes as k^2 for $k \to 0$, as one expect for Goldstone modes.

For $z > 0$ $\omega(k)$ behaves as an acoustical mode $\omega(k) \sim k$ in the range of small k, whereas in the limit $k \to \infty$ $\omega(k)$ tends to the ionization potential $\Delta(z)$.

In the region of intermediate values of k, when $kl \sim 1$, the dispersion relation develops the dips as z is increased. At certain critical value of $z = z_{\mathrm{cr}}$ the modes in the vicinity of the minima become equal to zero and are named as soft modes. Their appearance testify that the two-layer system undergo a phase transition to the Wigner crystal state.

The similar results concerning the linear and quadratic dependences of the dispersion relations in the range of small wave vectors q were obtained by Kuramoto and Horie,[32] who studied the coherent pairing of electrons and holes spatially separated by the insulator barrier in the structure of the type coupled double quantum wells (CDQW).

The magnetic field is sufficiently strong, so that the carriers populate only their lowest Landau levels (LLL) in the conduction and valence bands. Apparently the electron–hole interaction becomes less important than the repulsive electron–electron and hole–hole interactions as the separation z increases. However at low densities the ground state of the system will be the excitonic phase, instead of the Wigner lattice, for which the repulsive interaction is responsible. The reason is that the energy per electron–hole pair in the excitonic phase is lower than in Wigner crystal. The BEC of magnetoexcitons in the state with zero total momentum was considered and the dispersion relation of the collective excitation modes was derived. In the case $z \neq 0$ the lowest excitation branch has a linear dispersion relation in the region of small wave vectors q $\omega(q) \sim ql$; whereas at $z = 0$ it transforms in the quadratic dependence $\omega(q) \sim (ql)^2$; Kuramoto and Horie mentioned that the linear dispersion relation originates in the fact that at $z \neq 0$ the repulsive Coulomb interaction prevails and the carriers feel this resulting repulsive long-range force.[32] As in the Bogoliubov theory of weakly interacting Bose gas the repulsive interaction leads to the transformation of the quadratic dispersion relation into another one with the linear dependence at small wave vectors.

Spontaneous coherence in a two-component electron gas created in bilayer quantum well structure in a strong perpendicular magnetic field was recently studied experimentally by Eisenstein[33] and theoretically by MacDonald.[34]

The bilayer electron–electron system is much easy to realize in experiment than e–h bilayer, when the holes are created in the valence band and are spatially separated from the electrons in the conduction band. The experimental indications of spontaneous coherence have been seen first in e–e bilayer, which is analogous to Josephson junction. When the two 2D electron layers each at half-filling of the lowest Landau level (LLL) are sufficiently close together, then the ground state of the system possesses interlayer phase coherence. The ground state can be considered as an equilibrium Bose-Einstein condensate of excitons formed by the electrons on the LLL in the conduction band with the residence on one layer and the holes on the LLL of the conduction band with the residence on another layer. This collective state exhibits the quantum Hall effect when electrical currents are driven in parallel through two layers.[33] Counterflow transport experiments were realized. The oppositely directed currents were driven through the two layers. The counterflow proceeds via the collective transport of neutral particles, i.e. interlayer excitons. The Hall resistance of the individual layer vanishes at $T \to 0$ in the collective phase. A weak dissipation is present at finite temperatures. The free vortices are present at all temperatures being induced by the disorders. The existence of the anticipated Goldstone mode linearly dispersing was confirmed experimentally.[33] This mode is the consequence of a spontaneously broken $U(1)$ symmetry in the bilayer system. Measurements of the tunneling conductance between the layers have shown that the tunneling conductance at zero bias grows explosively, when the separation between the layers is brought below a critical value.[33]

The counterflow conductivity and inter-layer tunneling experiments both suggest that the system do not have long range order because of the presence of the unbound vortices nucleated by disorder. The finite phase coherence length appears.[34]

The appearance of the soft modes in the spectrum of the collective elementary excitations may signalize not only about the possible phase transition of the two-layer system to the Wigner crystal state or to the charge-density-wave (CDW) of a 2D electron system, but also to another variant of the excitonic charge-density-wave (ECDW) state. This new state was revealed theoretically by Chen and Quinn,[35,36] who studied the ground state and the collective elementary excitations of a system consisting of spatially separated electron and hole layers in strong magnetic field. When the interlayer Coulomb attraction is strong the electrons and holes pair together to form excitons. Excitonically condensed state of e–h pairs is the preferable ground state. If the layer separation is larger than a critical value, a novel excitonic-density-wave state is found to have a lower energy than either a homogeneous exciton fluid or a double charge-density-wave state in 2D electron system.

All these details and information will permit to better understand the results of our paper which is organized as follows.

In the section two the breaking of the gauge symmetry of the initial Hamiltonian is introduced by an alternative method following the idea proposed by Bogoliubov in his theory of quasiaverages.[37] The equivalence with another Bogoliubov u–v transformation method is revealed.

In the section three the equations of motion for the operators were obtained, whereas in section four on their base the main equations determining the many-particle Green's functions were deduced together with their truncation and the determination of the self-energy parts. The fifth section is devoted to collinear geometry and to analytical and numerical calculations of the relevant self energy parts. The sixth section is devoted to the conclusions.

2. THE BREAKING OF THE GAUGE SYMMETRY OF THE INITIAL HAMILTONIAN. TWO EQUIVALENT REPRESENTATIONS

For the very beginning we will introduce the operators describing the magneto-excitons and plasmons, and their commutation relations.

The creation and annihilation operators of magnetoexcitons are two-particle operators reflecting the electron–hole (e–h) structure of the excitons. They are denoted below as $d^{\dagger}(\vec{p})$ and $d(\vec{p})$, where $\vec{p}(p_x, p_y)$ is the two-dimensional wave vector. There are also the density fluctuation operators for electrons $\hat{\rho}_e(\vec{Q})$ and for holes $\hat{\rho}_h(\vec{Q})$ as well as their linear combinations $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$. They are determined below

$$\begin{aligned}
\hat{\rho}_e(\vec{Q}) &= \sum_t e^{iQ_y t l^2} a^{\dagger}_{t-(Q_x/2)} a_{t+(Q_x/2)} \\
\hat{\rho}_h(\vec{Q}) &= \sum_t e^{iQ_y t l^2} b^{\dagger}_{t+(Q_x/2)} b_{t-(Q_x/2)} \\
\hat{\rho}(\vec{Q}) &= \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q}) \\
\hat{D}(\vec{Q}) &= \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q}) \\
d^{\dagger}(\vec{P}) &= \frac{1}{\sqrt{N}} \sum_t e^{-iP_y t l^2} a^{\dagger}_{t+(P_x/2)} b^{\dagger}_{-t+(P_x/2)} \\
d(\vec{P}) &= \frac{1}{\sqrt{N}} \sum_t e^{iP_y t l^2} b_{-t+(P_x/2)} a_{t+(P_x/2)} \\
\hat{N}_e &= \hat{\rho}_e(0) \\
\hat{N}_h &= \hat{\rho}_h(0) \\
\hat{\rho}(0) &= \hat{N}_e - \hat{N}_h \\
\hat{D}(0) &= \hat{N}_e + \hat{N}_h
\end{aligned} \tag{1}$$

and are expressed through the Fermi creation and annihilation operators $a^{\dagger}_p$, a_p for electrons and $b^{\dagger}_p$, b_p for holes. The e–h Fermi operators depend on two quantum numbers. In Landau gauge one of them is the wave number p and the second one is the quantum number n of the Landau level. In the lowest Landau level (LLL) approximation n has only the value zero and its notation is dropped. The wave number p enumerates the N-fold degenerate states of the 2D electrons in a strong magnetic field. N can be expressed through the layer surface area S and the magnetic length l as follows: $N = (S/2\pi l^2)$; $l^2 = (\hbar c/eH)$, where H is the magnetic field strength. The operators (1) obey to the following commutation relations, most of which were discussed for first time in the papers[5, 13]

$$\begin{aligned}
[\hat{\rho}(\vec{Q}), \hat{\rho}(\vec{P})] &= -2iSin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{P}+\vec{Q}) \\
[\hat{D}(\vec{Q}), \hat{D}(\vec{P})] &= -2iSin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{P}+\vec{Q})
\end{aligned} \tag{2}$$

$$\begin{aligned}
[\hat{\rho}(\vec{Q}), \hat{D}(\vec{P})] &= -2iSin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{D}(\vec{P}+\vec{Q}) \\
[d(p), d^{+}(Q)] &= \delta_{kr}(\vec{P},\vec{Q}) - \frac{1}{N}\left[iSin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{P}-\vec{Q}) + Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{D}(\vec{P}-\vec{Q})\right] \\
[\hat{\rho}(\vec{P}), d(\vec{Q})] &= 2iSin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d(\vec{P}+\vec{Q}) \\
[\hat{\rho}(\vec{P}), d^{+}(\vec{Q})] &= -2iSin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^{+}(-\vec{P}+\vec{Q}) \\
[\hat{D}(\vec{P}), d^{+}(\vec{Q})] &= 2Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^{+}(\vec{Q}-\vec{P}) \\
[\hat{D}(\vec{P}), d(\vec{Q})] &= -2Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d(\vec{P}+\vec{Q})
\end{aligned} \tag{3}$$

One can observe that the density fluctuation operators (1) with different wave vectors $\vec{P}$ and $\vec{Q}$ do not commute. Their non-commutativity is related with the vorticity which accompanies the presence of the strong magnetic field and depends on the vector-product of two wave vectors $\vec{P}$ and $\vec{Q}$ and its projection on the direction of the magnetic field $[\vec{P}\times\vec{Q}]_z$. These properties considerably influence on the structure of the equations of motion for the operators (1) and determine new aspect of the magneto-exciton-plasmon physics. Indeed in the case of 3D e–h plasma in the absence of the external magnetic field the density fluctuation operators do commute.[38] The magneto-exciton creation and annihilation operators $d^{\dagger}(\vec{p})$ and $d(\vec{Q})$ as in general case do not obey exactly to the Bose commutation rule. Their deviation from it is proportional to the density fluctuation operators $\hat{\rho}(\vec{P}-\vec{Q})$ and $\hat{D}(\vec{P}-\vec{Q})$. The discussed above operators determine the structure of the 2D e–h system Hamiltonian in the LLL approximation. In thes previous papers[16, 17, 19–21] the initial Hamiltonian was gauge-invariant.

It has the form

$$\hat{H} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \hat{N}_e - \hat{N}_h] - \mu_e\hat{N}_e - \mu_h\hat{N}_h \tag{4}$$

where

$$W_{\vec{Q}} = \frac{2\pi e^2}{\varepsilon_0 S|\vec{Q}|}Exp\left[-\frac{Q^2 l^2}{2}\right]; \quad \mu = \mu_e + \mu_h \tag{5}$$

The energy of the two-dimensional magnetoexciton $E_{\text{ex}}(P)$ depends on the two-dimensional wave vector $\vec{P}$ and forms a band with the dependence

$$E_{\text{ex}}(\vec{P}) = -I_{\text{ex}}(\vec{P}) = -I_l + E(\vec{P})$$

$$
\begin{aligned}
&I_{ex}(\vec{P}) = I_l e^{-(P^2l^2/4)} I_0\left(\frac{P^2l^2}{4}\right)\\
&I_l = \frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}}; \quad \sum_{\vec{Q}} W_{\vec{Q}} = I_l
\end{aligned}
\tag{6}
$$

The ionization potential $I_{ex}(P)$ is expressed through the modified Bessel function $I_0(z)$, which has the limiting expressions.

$$
\underset{z\to 0}{I_0(z)} = 1 + \frac{z^2}{4} + \cdots; \quad \underset{z\to\infty}{I_0(z)} = \frac{e^z}{\sqrt{2\pi z}} \tag{7}
$$

It means that the function $E(P)$ can be approximated as follows

$$
\begin{aligned}
&\underset{P\to 0}{E(\vec{P})} = \frac{\hbar^2 P^2}{2M}; \quad M = M(0) = 2\sqrt{\frac{2}{\pi}}\frac{\hbar^2\varepsilon_0}{e^2 l}\\
&\underset{P\to\infty}{E(P)} = I_l\left(1 - \frac{\sqrt{2/\pi}}{Pl}\right); \quad l^2 = \frac{\hbar c}{eH}
\end{aligned}
\tag{8}
$$

Instead of the chemical potential μ (5) we will use the value $\bar{\mu}$ accounted from the bottom of the exciton band

$$
\bar{\mu} = \mu - E_{ex}(0) = \mu + I_l \tag{9}
$$

In the case of BEC of the magnetoexcitons on the state with $k \neq 0$ the chemical potential accounted from the exciton level $E_{ex}(k)$ will lead to the expression

$$
\mu - E_{ex}(\vec{K}) = \bar{\mu} - E(\vec{K}) \tag{10}
$$

To introduce the phenomenon of Bose-Einstein condensation (BEC) of excitons the gauge symmetry of the initial Hamiltonian was broken by the help of the unitary transformation $\hat{D}(\sqrt{N_{ex}})$ following the Keldysh-Kozlov-Kopaev method.[27] We can shortly remember the main outlines of the Keldysh-Kozlov-Kopaev method,[27,39] as it was realized in the papers.[20,21] The unitary transformation $\hat{D}(\sqrt{N_{ex}})$ was determined by the formula (8).[20] Here N_{ex} is the number of condensed excitons. It transforms the operators a_p, b_p to another ones α_p, β_p, as is shown in the formulas (13), (14),[20] and gives rise to the BCS-type wave function $|\psi_g(\bar{k})\rangle$ of the new coherent macroscopic state represented by the expression (10).[20] These results are summarized below

$$
\begin{aligned}
&\hat{D}(\sqrt{N_{ex}}) = \exp\left[\sqrt{N_{ex}}(d^\dagger(\bar{k}) - d(\bar{k}))\right]\\
&|\psi_g(\bar{k})\rangle = \hat{D}(\sqrt{N_{ex}})|0\rangle\\
&\alpha_p = \hat{D}a_p\hat{D}^\dagger = u a_p - \mathrm{v}\left(p - \frac{k_x}{2}\right)b^\dagger_{k_x-p}\\
&\beta_p = \hat{D}b_p\hat{D}^\dagger = u b_p + \mathrm{v}\left(\frac{k_x}{2} - p\right)a^\dagger_{k_x-p}\\
&a_p = u\alpha_p + \mathrm{v}\left(p - \frac{k_x}{2}\right)\beta^\dagger_{k_x-p}\\
&b_p = u\beta_p - \mathrm{v}\left(\frac{k_x}{2} - p\right)\alpha^\dagger_{k_x-p}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&a_p|0\rangle = b_p|0\rangle = 0; \quad \alpha_p|\psi_g(\bar{k})\rangle = \beta_p|\psi_g(\bar{k})\rangle = 0\\
&u = \cos g; \quad \mathrm{v} = \sin g; \quad \mathrm{v}(t) = \mathrm{v}e^{-ik_y t l^2}\\
&g = \sqrt{2\pi l^2 n_{ex}}; \quad n_{ex} = \frac{N_{ex}}{S} = \frac{\mathrm{v}^2}{2\pi l^2} \quad g = \mathrm{v}; \quad v = Sinv
\end{aligned}
\tag{12}
$$

The developed theory[20,21] is true in the limit $\mathrm{v}^2 \approx Sin^2\mathrm{v}$, what means the restriction $\mathrm{v}^2 < 1$. In the frame of this approach the collective elementary excitations can be studied constructing the Green's functions on the base of operators α_p, β_p and having deal with the transformed cumbersome Hamiltonian $\hat{\hat{\mathscr{H}}} = D(\sqrt{N_{ex}})\hat{H}D^\dagger(\sqrt{N_{ex}})$.

We propose another way, which is supplementary but completely equivalent to the previous one and is based on the idea suggested by Bogoliubov in his theory of quasiaverages.[37] Considering the case of a 3D ideal Bose gas with the Hamiltonian

$$
H = \sum_{\vec{p}}\left(\frac{\hbar^2 p^2}{2m} - \mu\right)a^\dagger_{\vec{p}}a_{\vec{p}} \tag{13}
$$

where a^+_p, a_p are Bose operators and μ is the chemical potential, Bogoliubov added a term

$$
-\eta\sqrt{V}(a_0 e^{i\varphi} + a_0 e^{-i\varphi}) \tag{14}
$$

breaking the gauge symmetry and proposed to consider the BEC on the state with $p = 0$ in the frame of the Hamiltonian

$$
\hat{\mathscr{H}} = \sum_p\left(\frac{\hbar^2 p^2}{2m} - \mu\right)a^\dagger_p a_p - \eta\sqrt{V}(a^\dagger_0 e^{i\varphi} + a_0 e^{-i\varphi}) \tag{15}
$$

where

$$
\eta = -\mu\sqrt{\frac{N_0}{V}} = -\mu\sqrt{n_0}; \quad -\frac{\eta}{\mu} = \sqrt{n_0} \tag{16}
$$

We will name the Hamiltonian of the type (15) as the Hamiltonian of the theory of quasiaverages. It is written in the frame of the operators a^+_p, a_p of the initial Hamiltonian (13).

Our intention is to apply this idea to the case of BEC of interacting 2D magnetoexcitons and to deduce explicitly the Hamiltonian of the type (15) with the finite parameters μ and η but with the ralation of the type (16). We intend to formulate the new Hamiltonian with broken symmetry in the terms of he operators a_p, b_p avoiding the obligatory crossing to the operators α_p, β_p (11) at least at some stages of the investigation were the representation in the a_p, b_p operators remains preferential.

Of course the two representations are completely equivalent and complimentary each other. We will follow the quasiaverage variant (15) instead of u, v variant (11–12), because it opens some new possibilities, which were not studied up till now to the best of our knowledge. For example the Hamiltonian of the type (15) is more simple than the Hamiltonian $\hat{\hat{\mathscr{H}}} = D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$ in the α_p, β_p representation and the deduction of the equation of motion for the operators (18) and for the many-particle Green's functions constructed on their base is also much simple. We will profit by this advantage at some stages of investigation. On the contrary, when we will have deal with the calculations of the average values of different operators on the base of the ground coherent macroscopic state (11) or using the coherent excited states, as we have done in the papers,[20,21] the most convenient way is to use the α_p, β_p representation. We will use in the wide manner the both representations. The new variant in the style of the theory of quasiaverages can be realized rewriting the transformed Hamiltonian $D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$ in the a_p, b_p representation as folllows below. To demonstrate it we will represent the unitary transformation

$$\hat{D}(\sqrt{N_{ex}}) = e^{\hat{X}} = \sum_{n=0}^{\infty}\frac{\hat{X}^n}{n!} \qquad D^{\dagger}(\sqrt{N_{ex}}) = e^{-\hat{X}} \tag{17}$$

where

$$\hat{X} = \sqrt{N_{ex}}(e^{i\varphi}d^{\dagger}(K) - e^{-i\varphi}d(K)) \qquad \hat{X}^{\dagger} = -\hat{X} \tag{18}$$

The creation and annihilation operators $d^{+}(k)$, $d(k)$ (18) are written in the Landau gauge when the electrons and holes forming the magnetoexcitons are situated on their lowest Landau levels (LLL). Only this variant is considered here without taking into account of the excited Landau levels (ELL), as it was done in.[20] The BEC of 2D magnetoexcitons is considered on the single-exciton state characterized by two-dimensional wave vector $\vec{k}$. Expanding in series the unitary operators $D(\sqrt{N_{ex}})$, $D^{\dagger}(\sqrt{N_{ex}})$ we will find the transformed operator $\hat{\hat{\mathscr{H}}}$ in the form

$$\hat{\hat{\mathscr{H}}} = e^{\hat{X}}\hat{H}e^{-\hat{X}} = \hat{H} + \frac{1}{1!}[\hat{X},\hat{H}] + \frac{1}{2!}[\hat{X},[\hat{X},\hat{H}]] + \frac{1}{3!}[\hat{X},[\hat{X},[\hat{X},\hat{H}]]] + \cdots = \hat{\mathscr{H}} + \hat{\mathscr{H}}' \tag{19}$$

Here the Hamiltonian $\hat{\mathscr{H}}$ contains the main contributions of the first two terms in the series expansion (19), whereas the operator $\hat{\mathscr{H}}'$ gathers the all remaining terms.

As one can see looking at the formulas (18) operator $\hat{X}$ is proportional to the square root of the exciton concentration $\sqrt{N_{ex}}$, which is proportional to the filling number v. One can see that the contributions arising from the first commutator $[\hat{X}, \hat{H}]$ are proportional to v, the contributions arising from the second commutator $[\hat{X}, [\hat{X}, \hat{H}]]$ are proportional to v^2 and so on. Following the Bogoliubov's theory of quasiaverages only the linear terms of the type $(d^{+}(k)e^{i\varphi} + e^{-i\varphi}d(k))\nu$ arising from the first commutator $[\hat{X}, \hat{H}]$ must be included into $\hat{\mathscr{H}}$.

The Hamiltonian $\hat{\mathscr{H}}$ with the broken gauge symmetry describing the BEC of 2D magnetoexcitons on the state with wave vector $k \neq 0$ being written in the style of the Bogoliubov's theory of quasiaverages has the form

$$\hat{\mathscr{H}} = \hat{H} + \sqrt{N_{ex}}(\bar{\mu} - E(\vec{K}))(e^{i\varphi}d^{\dagger}(\vec{K}) + e^{-i\varphi}d(\vec{K})) \tag{20}$$

For the case of an ideal 2D Bose gas we can rewrite the coefficient $-\eta\sqrt{V}$ in the Hamiltonian (15), in the form $-\eta\sqrt{N}$ and comparing it with the deduced expression (20), we will find

$$\eta = (E(k) - \bar{\mu})\text{v} \tag{21}$$

where N and the filling number v are determined by the expressions (1) and (12). The relation (21) coincides exactly with the relation (16) of the Bogoliubov's theory of quasiaverages. In the case of ideal Bose gas η and $(E(k) - \bar{\mu})$ both tend to zero, whereas the filling number is real and different from zero. In the interacting exciton gas the parameter η and $(E(k) - \bar{\mu})$ are both different from zero.

Now the remaining terms gathered in $\hat{\mathscr{H}}'$ will be written. They contain the contributions proportional to v^2, v^3 and so on. There is also one term proportional to v, but it is nonlinear containing the products of the exciton and fluctuation density operators. Their influence on the BEC of magnetoexcitons is less in comparison with the second term in the expression (20). The first terms included in $\hat{\mathscr{H}}'$ are

$$\begin{aligned}\hat{\mathscr{H}}' = &-(2i)\sqrt{N_{ex}}\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \\ &\times (e^{i\varphi}d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q}) - e^{-i\varphi}\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})) \\ &+ 2N_{ex}\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \\ &\times \left(e^{2i\varphi}d^{\dagger}(\vec{K}-\vec{Q})d^{\dagger}(\vec{K}+\vec{Q}) + e^{-2i\varphi}d(\vec{K}+\vec{Q}) \right.\\ &\times d(\vec{K}-\vec{Q}) + 2d^{\dagger}(\vec{K}-\vec{Q})d(\vec{K}-\vec{Q}) \\ &\left. - \frac{1}{N}\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right) \\ &+ N_{ex}(E(k) - \bar{\mu})\left(1 - \frac{\hat{D}(0)}{N}\right) \\ &- i\frac{N_{ex}}{N}\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) Cos\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \\ &\times (\hat{D}(\vec{Q})\hat{\rho}(-\vec{Q}) - \hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})) + \cdots\end{aligned} \tag{22}$$

Below we will construct the equations of motion for the operators (1) on the base of the Hamiltonian (20) in the quasiaverages theory approximation (QATA).

3. THE EQUATION OF MOTION FOR OPERATORS. THE QUANTA OF COULOMB ENERGY RELATED WITH VORTICITY OF THE MAGNETIC FIELD

The starting Hamiltonian $\hat{\mathscr{H}}$ in QATA has form

$$\hat{\mathscr{H}} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}[\rho(\vec{Q})\rho(-\vec{Q}) - \hat{N}_e - \hat{N}_h] - \mu_e\hat{N}_e - \mu_h\hat{N}_h - \eta\sqrt{N}(e^{i\varphi}d^{\dagger}(k) + e^{-i\varphi}d(k)) \tag{23}$$

The equations of motion for the operators (1) are obtained using the commutation relations (3). They are

$$\begin{aligned}
i\hbar\frac{d}{dt}d(\vec{P}) &= [d(\vec{P}), \hat{\mathscr{H}}] \\
&= (E(\vec{P}) - \bar{\mu})d(\vec{P}) - \eta\sqrt{N}e^{i\varphi}\delta_{kr}(\vec{P},\vec{K}) \\
&\quad - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q}) \\
&\quad + \eta e^{i\varphi}\left[iSin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\frac{\hat{\rho}(\vec{P}-\vec{K})}{\sqrt{N}} + Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\frac{\hat{D}(\vec{P}-\vec{K})}{\sqrt{N}}\right]
\end{aligned}$$

$$\begin{aligned}
i\hbar\frac{d}{dt}d^{\dagger}(2\vec{K}-\vec{P}) &= [d^{\dagger}(2\vec{K}-\vec{P}), \hat{\mathscr{H}}] \\
&= (\bar{\mu} - E(2\vec{K}-\vec{P}))d^{\dagger}(2\vec{K}-\vec{P}) + \eta\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec{P},\vec{K}) \\
&\quad - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q}) \\
&\quad \times\hat{\rho}(-\vec{Q}) - \eta e^{-i\varphi}\left[iSin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\frac{\hat{\rho}(\vec{P}-\vec{K})}{\sqrt{N}} + Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\frac{\hat{D}(\vec{P}-\vec{K})}{\sqrt{N}}\right]
\end{aligned} \tag{24}$$

$$\begin{aligned}
i\hbar\frac{d}{dt}\hat{\rho}(\vec{P}-\vec{K}) &= [\hat{\rho}(\vec{P}-\vec{K}), \hat{\mathscr{H}}] \\
&= -i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right) \\
&\quad \times[\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})\hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})] \\
&\quad - 2i\eta\sqrt{N}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)[e^{-i\varphi}d(\vec{P}) - e^{i\varphi}d^{\dagger}(2\vec{K}-\vec{P})]
\end{aligned}$$

$$\begin{aligned}
i\hbar\frac{d}{dt}\hat{D}(\vec{P}-\vec{K}) &= [\hat{D}(\vec{P}-\vec{K}), \hat{\mathscr{H}}] \\
&= -i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right) \\
&\quad \times[\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{K}-\vec{Q}) + \hat{D}(\vec{P}-\vec{K}-\vec{Q})\hat{\rho}(\vec{Q})] \\
&\quad + 2\eta\sqrt{N}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)[e^{-i\varphi}d(\vec{P}) - e^{i\varphi}d^{\dagger}(2\vec{K}-\vec{P})]
\end{aligned}$$

Here

$$\eta = (E_{\text{ex}}(K) - \mu)\text{v} = (E(K) - \bar{\mu})\text{v}; \quad \upsilon = \text{v}^2; \quad N_{\text{ex}} = \text{v}^2 N \tag{25}$$

Now we must pay attention to one important aspect of these equations of motion, closely related with the noncommutativity of the operators (1) expressed by the formulas (2) and (3). Applying them one can prove the equivalent expressions for the exciton operator $d(P)$

$$\begin{aligned}
&(E(P) - \bar{\mu})d(P) - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\rho(\vec{Q})d(\vec{P}-\vec{Q}) \\
&= -\bar{\mu}d(P) - i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right) \\
&\quad \times[\rho(\vec{Q})d(\vec{P}-\vec{Q}) + d(\vec{P}-\vec{Q})\rho(\vec{Q})] \\
&= -(\bar{\mu} + E(P))d(P) \\
&\quad - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d(\vec{P}-\vec{Q})\rho(\vec{Q}) = \cdots
\end{aligned} \tag{26}$$

as well as for the density fluctuation operator $\hat{\rho}(P)$

$$\begin{aligned}
&-i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right) \\
&\quad \times[\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}) + \hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})] \\
&= E(P)\hat{\rho}(P) - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}) \\
&= -E(P)\hat{\rho}(P) \\
&\quad - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q}) = \cdots
\end{aligned} \tag{27}$$

They can be verified taking into account the relation

$$2\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right) = E(P) \tag{28}$$

The quantum of the Coulomb energy $E(P)$ is related with the vorticity existing in the frame of electron–hole (e–h) system in the presence of a strong perpendicular magnetic field. A full spectrum of these quanta with arbitrary wave vector $\vec{P}$ does exist not only in the case of e–h system, but also in the case of pure electron or pure hole systems. We are supposing that their origin is related with the existence of N magnetic flux quanta $\phi_0 = hc/e$ discussed in Ref. [3] in the case of FQHE. The flux quanta enforce the creation of N vortices in the 2DEG, lead to the creation of composite fermions and bosons, accompanying the transport phenomena.[3] The unlimited reservoir of energy created by the magnetic field is characterized by energy quanta $E(P)$, which depend only on the square electric charge e^2 and magnetic length l and does not depend on the e–h densities. They could be named as vortex or spiral Coulomb energy quanta. In our previous paper[40] they were named as plasmon quanta, but on our opinion it is better to conserve the name of plasmon quanta to the intra-Landau level excitations whose energy depend on the filling factors.

As follows from the equalities (27), (28) the induced by vortices the Coulomb energy quanta can be added or substracted as a free part terms outside the nonlinear terms, if we will change simultaneously the corresponding nonlinear terms.

In the case of matter interacting with the resonant laser radiation with the frequency ω_L in the rotating reference frame the energy of quasiparticles is changed by the photon quantum energy $\hbar\omega_L$. Such type of energy which appears in the case of unlimited reservoir of energy is named as quasi-energy.[41] The new supplementary quasi-energy branches give rise to many effects gathered by the common name as optical Stark effect.[39] On our opinion something similar takes place in the presence of a strong magnetic field, but in difference on the laser radiation with a well defined frequency $\omega_L = ck_0$ and wave vector k_0, in the case of a strong magnetic field there are a large spectra of frequencies and wave vectors. Adding or extracting the quanta $E(P)$ we can form many virtual complexes of quasiparticles with different free energies. They can be named as quasi-energy complexes. As we will see below the most of them will have great damping rates and will be physical meaningless. The choosing of the concrete forms of equations of motion depends in great manner on the theoretical methods, which we intend to apply.

We will apply below the Green's function method. In this case the free energy terms in the equation of motion for operators as usual play the role of the proper energies in zero order approximation. They can determine the zero-order Green's function, whereas the nonlinear terms can be taken into account in higher order of the perturbation theory. Of course, when the equations of motion for the Green's function are treated exactly in this case it is indifferent which starting variant was selected, because all of them are completely equivalent. But in reality it is impossible to solve exactly the infinite chains of equations of motion for Green's functions and some concrete approximations are inevitable.

Taking into account these considerations we have chosen the equations of motion for the exciton creation and annihilation operations $d^\dagger(P)$, $d(P)$ with a free energy term accounted from the exciton chemical potential in the form $(E_{ex}(P) - \mu)$. $E_{ex}(P)$ coincides with the energy of the magnetoexciton without any corrections depending on the exciton–exciton interaction, what means without concentration corrections. The equations of motion for the density fluctuation operators $\rho(\vec{P})$ and $D(\vec{P})$ were chosen in the first variant of the Eq. (27) without free energy terms, because the proper energies of the intra-lowest Landau level excitations depend on the filling factors and can not be represented by quanta $E(P)$ in any forms. The true expressions for the plasmon eigenenergies will appear in the second order of the perturbation theory developed on concentration parameter, and its value will depend on $v^2(1 - v^2)$. Another important consideration for the selection of the starting equations of motion having in view the Green's function method, is the damping rates of the obtained elementary excitations. The imaginary parts of the eigenenergies of the elementary excitations depend on the real Coulomb scattering processes with the participations of the quasiparticles as well as on their free energies which appear in zero order approximation. In most cases the damping rates are of the same order of magnitude as the corresponding real parts due the absence of small parameter related with Coulomb energy. It means that such elementary excitations can not exist and have not any physical meaning.

Once again we can underline that it happens because the Coulomb interaction energy can not be considered as a small perturbation. In fact there is a unique possibility to chose the equations of motion for the operators $\rho(\vec{P})$ and $D(\vec{P})$ as it was realized in our equations of motion. One can represent different variants of equations of motion with different free energy terms as corresponding to different quasienergy complexes consisting from quasiparticles and vortex Coulomb quanta $E(P)$. This suggestion is supported and induced by the well known concept of composite particles created by electrons and magnetic flux quanta ϕ_0 [3] and by the supposition that their existence must be evidenced also in another phenomena not so far from the FQHE. But trying to do it, and verifying the consequences posteriori we arrived to the conclusion that most of them have great damping rates and do not exist. The unique possibility to obtain in the frame of the Green's function method the intra-LLL excitations of the plasmon type without damping at all in our approximation is the variant chosen by us and written above in the frame of the equations of motion (24). But for the magnetoexcitons some different quasienergy complexes are possible. Here we will discuss only the variant with an usual dispersion law. On the base of equations of motion (24) the Green's functions will be introduced and the chains of equations of motion for the Green's functions will be developed.

4. MANY-OPERATOR, MANY-PARTICLE GREEN'S FUNCTIONS

Following the equations of motion (24) we will introduce four interconnected retarded Green's functions at $T = 0$ [42, 43]

$$\begin{aligned} G_{11}(\vec{P}, t) &= \langle\langle d(\vec{P}, t); d^{\dagger}(\vec{P}, 0)\rangle\rangle \\ G_{12}(\vec{P}, t) &= \langle\langle d^{\dagger}(2\vec{K}-\vec{P}, t); d^{\dagger}(\vec{P}, 0)\rangle\rangle \\ G_{13}(\vec{P}, t) &= \left\langle\left\langle \frac{\hat{\rho}(\vec{P}-\vec{K}, t)}{\sqrt{N}}; d^{\dagger}(\vec{P}, 0)\right\rangle\right\rangle \\ G_{14}(\vec{P}, t) &= \left\langle\left\langle \frac{\hat{D}(\vec{P}-\vec{K}, t)}{\sqrt{N}}; d^{\dagger}(\vec{P}, 0)\right\rangle\right\rangle \end{aligned} \tag{29}$$

They are determined by the relations

$$\begin{aligned} G(t) &= \langle\langle \hat{A}(t); \hat{B}(0)\rangle\rangle = -i\theta(t)\langle[A(t), B(0)]\rangle \\ \hat{A}(t) &= e^{(iHt/\hbar)}\hat{A}e^{-(iHt/\hbar)} \\ [\hat{A}, \hat{B}] &= \hat{A}\hat{B} - \hat{B}\hat{A} \end{aligned} \tag{30}$$

where $\hat{H}$ is the Hamiltonian (23).

The average $\langle\rangle$ will be calculated at $T = 0$ in HFB approximation using the ground state wave function $|\psi_g(k)\rangle$ (11). The time derivative of the Green's function is calculated as follows

$$\begin{aligned} i\hbar\frac{d}{dt}G(t) &= i\hbar\frac{d}{dt}\langle\langle A(t); B(0)\rangle\rangle \\ &= \hbar\delta(t)\langle[\hat{A}(0), \hat{B}(0)]\rangle + \left\langle\left\langle i\hbar\frac{d}{dt}A(t); B(0)\right\rangle\right\rangle \\ &= \hbar\delta(t)C + \langle\langle[\hat{A}(t), \hat{H}]; \hat{B}(0)\rangle\rangle \end{aligned} \tag{31}$$

By C will be denoted the average values, which do not depend on t. They are not needed in an explicit form for the determination of the energy spectrum of the elementary excitations.

Fourier transforms of the Green's functions (29) will be denoted as

$$\begin{aligned} G_{11}(\vec{P}, \omega) &= \langle\langle d(\vec{P}) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\ G_{12}(\vec{P}, \omega) &= \langle\langle d^{\dagger}(2\vec{K}-\vec{P}) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\ G_{13}(\vec{P}, \omega) &= \left\langle\left\langle \frac{\hat{\rho}(\vec{P}-\vec{K})}{\sqrt{N}} \middle| d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega} \\ G_{14}(\vec{P}, \omega) &= \left\langle\left\langle \frac{\hat{D}(\vec{P}-\vec{K})}{\sqrt{N}} \middle| d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega} \end{aligned} \tag{32}$$

The two representations are related each-other

$$G(\vec{P}, \omega) = \int_{-\infty}^{\infty} e^{i\omega t}G(\vec{P}, t)\, dt = \int_{0}^{\infty} e^{i\omega t-\delta t}G(\vec{P}, t)\, dt$$

where the infinitesimal value $\delta \to +0$ guarantees for the retarded Green's function $G(\vec{P}, t)$ the convergence of the integral in the interval $(0, \infty)$.

The equation of motion in the frequency representation can be deduced on the base of Eq. (31)

$$\begin{aligned} &\int_{-\infty}^{\infty} dt e^{i\omega t} i\hbar\frac{dG(t)}{dt} \\ &= i\hbar\int_{0}^{\infty} dt e^{i\omega t-\delta t}\frac{dG(t)}{dt} \\ &= -i\hbar\int_{0}^{\infty} dt G(t)\frac{de^{i\omega t-\delta t}}{dt} \\ &= (\hbar\omega + i\delta)G(\omega) \\ &= C + \int_{-\infty}^{\infty} dt\langle\langle[\hat{A}(t), \hat{H}]; \hat{B}(0)\rangle\rangle e^{i\omega t} \end{aligned} \tag{33}$$

The Green's functions (32) will be named as one-operator Green's functions because they contain in the left hand side of the vertical line only one summary operator of the types $d(P)$, $d^{\dagger}(P)$, $\hat{\rho}(P)$ and $\hat{D}(P)$. At the same time these Green's functions are two-particle Green's functions, because the summary operators (1) are expressed through the products of two Fermi operators. In this sense the Green's functions (32) are equivalent with the two-particle Green's functions introduced by Keldysh and Kozlov in their fundamental paper,[27] forming the base of the theory of high density excitons in the electron–hole description. But in difference on Ref. [27] we are using the summary operators (1), which represent integrals on the wave vectors of relative motions.

The equations of motion for the Green's function (32) are the following

$$\begin{aligned} &[\hbar\omega + \bar{\mu} - E(\vec{P}) + i\delta]G_{11}(\vec{P}, \omega) \\ &= C - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\langle\langle\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q}) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\ &\quad + \eta e^{i\varphi}\left[iSin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)G_{13}(\vec{P}, \omega)\right. \\ &\quad \left. + Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)G_{14}(\vec{P}, \omega)\right] \\ &[\hbar\omega - \bar{\mu} + E(2\vec{K}-\vec{P}) + i\delta]G_{12}(\vec{P}, \omega) \\ &= C - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times\langle\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q}) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\ &\quad - \eta e^{-i\varphi}\left[iSin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)G_{13}(\vec{P}, \omega)\right. \\ &\quad \left. + Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)G_{14}(\vec{P}, \omega)\right] \end{aligned} \tag{34}$$

$$[\hbar\omega+i\delta]G_{13}(\vec{P},\omega)$$
$$=C-i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times\left\langle\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\right]\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$-2i\eta Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)[e^{-i\varphi}G_{11}(\vec{P},\omega)-e^{i\varphi}G_{12}(\vec{P},\omega)]$$

$$[\hbar\omega+i\delta]G_{14}(\vec{P},\omega)$$
$$=C-i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times\left\langle\left\langle\left[\hat{\rho}(\vec{Q})\frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}+\frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q})\right]\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$+2\eta Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)[e^{-i\varphi}G_{11}(\vec{P},\omega)-e^{i\varphi}G_{12}(\vec{P},\omega)]$$

The equation of motion (34) for one-operator Green's functions $G_{1j}(\vec{P},\omega)$, where $j=1,2,3,4$, give rise to new two-operator (four-particle) Green's functions of the types

$$\langle\langle\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$\langle\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$\left\langle\left\langle\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q})\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$

and

$$\left\langle\left\langle\frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q})\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$

generated by the nonlinear terms in the equations of motion (24) for the operators (1). It is a well known situation described by Zubarev[42] in his review article. For these two-operator Green's functions of the first generation following the rule (33) the new equations of motion were deduced. This second step in the frame of the given method will form the second link of an infinite chain of equations of motion. Both links constructed in such a way will be exact in the frame of the Hamiltonian (23). These new equations of motion will contain in their components new types of three-operator Green's functions of the first generation as well as new types of the two-operator Green's functions of the second generation, and so on. It can be demonstrated, for example, calculating the concrete two-operator Green's functions arising in the component of the equation of motion (34) for the starting Green's function $G_{13}(\vec{P},\omega)$. The obtained result is:

$$[\hbar\omega+i\delta]\left\langle\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\right]\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$=C+\left\langle\left\langle\left[\left(\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\right),\mathscr{H}\right]\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$=C-i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[(\vec{P}-\vec{K}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)$$
$$\times\left\langle\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{R})+\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\right]\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$-i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)$$
$$\times\left\langle\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})+\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{R})+\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}+\hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\right]\right|d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$-2i\eta Sin\left(\frac{[(\vec{P}-\vec{Q})\times\vec{K}]_z l^2}{2}\right)$$
$$\times\langle\langle[e^{-i\varphi}(d(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q}))-e^{i\varphi}(d^{\dagger}(2\vec{K}-\vec{P}+\vec{Q})\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})d^{\dagger}(2\vec{K}-\vec{P}+\vec{Q}))]\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$-2i\eta Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)$$
$$\times\langle\langle[e^{-i\varphi}(\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})d(\vec{Q}+\vec{K})+d(\vec{Q}+\vec{K})\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}))-e^{i\varphi}(\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})d^{\dagger}(\vec{K}-\vec{Q})+d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}))]\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \quad (35)$$

As we mentioned above, the three-operator Green's functions of the first generation have appeared, being accompanied by the new two-operator Green's functions of the second generation. Exact the same evolution of the equations of motion takes place for the all four starting

Green's functions $G_{1j}(\vec{P},\omega)$. The interruption of these infinite chains of equations of motion is needed using reasonable approximations. Following the Zubarev's method[43] we will truncate the three-operator Green's functions expressing them through the one-operator Green's functions (32) multiplied by the average values of another two remaining operators. This method, if applied to the two-operator Green's functions of the second generation means in fact to linearize them.

The linearization can be achieved conserving only the macroscopic large values of the operators substituting them by their average values at some well definite values of the wave vector and neglecting all their infinitesimal values as follows

$$
\begin{aligned}
d(\vec{P}) &\simeq \delta_{kr}(\vec{P},\vec{K})e^{i\varphi}\sqrt{N_{\text{ex}}}\\
d^{\dagger}(\vec{P}) &\simeq \delta_{kr}(\vec{P},\vec{K})e^{-i\varphi}\sqrt{N_{\text{ex}}}\\
\hat{D}(\vec{P}) &\simeq \delta_{kr}(\vec{P},0)\langle\hat{D}(0)\rangle \simeq \delta_{kr}(\vec{P},0)2N_{\text{ex}}\\
\rho(\vec{P}) &\simeq \delta_{kr}(\vec{P},0)\langle\hat{\rho}(0)\rangle = 0
\end{aligned}
\tag{36}
$$

The truncation procedure was successful applied in the case of electron–phonon interaction not only for the metals in normal states, but also for the superconductors.

It can be applied also in the case of Bose-Einstein condensed magnetoexcitons because this phenomenon was taken into account for the very beginning by the Bogoliubov method of quasiaverages. The calculations of the average values of the products of two operators exctracted from the left-hand side of the three-operator Green's functions will be made using the ground state wave function of the Bose-Einstein condensed magnetoexcitons. On this base some supplementary simplifications of the cumbersome expressions will be proposed.

Introducing the Green's function (35) into the sum on $\vec{Q}$ entering into the Eq. (34) for $G_{13}(\vec{P},\omega)$ we will obtain

$$
\begin{aligned}
&-i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times\left\langle\!\!\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\right]\middle| d^{\dagger}(\vec{P})\right\rangle\!\!\right\rangle_{\omega}\\
&= C - \frac{1}{(\hbar\omega+i\delta)}\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}}W_{\vec{R}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin\left(\frac{[(\vec{P}-\vec{K}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\\
&\quad\times\left\langle\!\!\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{Q})\right.\right.\right.\\
&\qquad + \hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{R})\\
&\qquad \left.\left.\left.+ \hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\right]\middle| d^{\dagger}(\vec{P})\right\rangle\!\!\right\rangle_{\omega}\\
&\quad - \frac{1}{(\hbar\omega+i\delta)}\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}}W_{\vec{R}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\\
&\quad\times\left\langle\!\!\left\langle\left[\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R}) + \frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{R})\right.\right.\right.\\
&\qquad + \hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\\
&\qquad \left.\left.\left.+ \hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{R})\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\right]\middle| d^{\dagger}(\vec{P})\right\rangle\!\!\right\rangle_{\omega}\\
&\quad - \frac{2\eta}{(\hbar\omega+i\delta)}\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin\left(\frac{[(\vec{P}-\vec{Q})\times\vec{K}]_z l^2}{2}\right)\\
&\quad\times[e^{-i\varphi}\langle\langle(d(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}\\
&\qquad - e^{i\varphi}\langle\langle(d^{\dagger}(2\vec{K}-\vec{P}+\vec{Q})\hat{\rho}(\vec{Q})\\
&\qquad\qquad + \hat{\rho}(\vec{Q})d^{\dagger}(2\vec{K}-\vec{P}+\vec{Q})) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}]\\
&\quad - \frac{2\eta}{(\hbar\omega+i\delta)}\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\\
&\quad\times[e^{-i\varphi}\langle\langle(\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})d(\vec{Q}+\vec{K})\\
&\qquad\qquad + d(\vec{Q}+\vec{K})\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}\\
&\qquad - e^{i\varphi}\langle\langle(\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})d^{\dagger}(\vec{K}-\vec{Q})\\
&\qquad\qquad + d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})) \mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}]
\end{aligned}
\tag{37}
$$

In the frame of the approximation (36) the two last sums in (37) happen to be equal to zero, due to the vorticity and symmetry properties of the system. But in the another similar cases the different from zero terms will appear. The truncations and the decouplings of the three-operator Green's functions generated by all four equations of motion (34) will be effectuated using the approximations

$$\left\langle\left\langle\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q})\,\middle|\,d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$\approx G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{Q},\vec{P}-\vec{K})\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\rangle + (\delta_{kr}(\vec{R},-\vec{Q})+\delta_{kr}(\vec{R},\vec{P}-\vec{K}))\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle]$$

$$\left\langle\left\langle\frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\,\middle|\,d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$\approx G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{Q},0)\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\rangle + (\delta_{kr}(\vec{R},\vec{P}-\vec{K})+\delta_{kr}(\vec{R},\vec{Q}+\vec{K}-\vec{P}))\times\langle\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})\hat{\rho}(\vec{Q}+\vec{K}-\vec{P})\rangle]$$

$$\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}}\,\middle|\,d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$\approx \delta_{kr}(\vec{Q},0)G_{14}(\vec{P},\omega)\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\rangle + G_{13}(\vec{P},\omega)(\delta_{kr}(\vec{R},\vec{P}-\vec{K})+\delta_{kr}(\vec{R},\vec{Q}+\vec{K}-\vec{P}))\times\langle\hat{\rho}(\vec{Q}+\vec{K}-\vec{P})\hat{D}(\vec{P}-\vec{K}-\vec{Q})\rangle$$

$$\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q})\frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q}-\vec{R})}{\sqrt{N}}\,\middle|\,d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}$$
$$\approx \delta_{kr}(\vec{R},-\vec{Q})G_{14}(\vec{P},\omega)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle + G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{R},\vec{P}-\vec{K})\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\rangle + \delta_{kr}(\vec{Q},\vec{P}-\vec{K})\langle\hat{\rho}(\vec{R})\hat{D}(-\vec{R})\rangle]$$

$$\langle\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})d(\vec{P}-\vec{Q}-\vec{R})\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$\approx \delta_{kr}(\vec{R},-\vec{Q})G_{11}(\vec{P},\omega)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle + G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{R},\vec{P}-\vec{K})\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})\rangle + \delta_{kr}(\vec{Q},\vec{P}-\vec{K})\langle\hat{\rho}(\vec{R})d(\vec{K}-\vec{R})\rangle]\sqrt{N}$$

$$\langle\langle\hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{R})d(\vec{P}-\vec{Q})|d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$\approx \delta_{kr}(\vec{Q},0)G_{11}(\vec{P},\omega)\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\rangle + G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{R},\vec{P}-\vec{K})+\delta_{kr}(\vec{R},\vec{Q}+\vec{K}-\vec{P})]\times\langle\hat{\rho}(\vec{Q}+\vec{K}-\vec{P})d(\vec{P}-\vec{Q})\rangle\sqrt{N}$$

$$\langle\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q}-\vec{R})\hat{\rho}(-\vec{R})\hat{\rho}(-\vec{Q})\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$\approx G_{12}(\vec{P},\omega)\delta_{kr}(\vec{R},-\vec{Q})\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle + G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{R},\vec{K}-\vec{P})\langle d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})\rangle + \delta_{kr}(\vec{Q},\vec{K}-\vec{P})\langle d^{\dagger}(\vec{K}-\vec{R})\hat{\rho}(-\vec{R})\rangle]\sqrt{N}$$

$$\langle\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q}-\vec{R})\hat{\rho}(\vec{R})\mid d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$$
$$\approx \delta_{kr}(\vec{Q},0)G_{12}(\vec{P},\omega)\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\rangle + G_{13}(\vec{P},\omega)[\delta_{kr}(\vec{R},\vec{P}-\vec{K})+\delta_{kr}(\vec{R},\vec{K}-\vec{P}-\vec{Q})]\times\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\rangle\sqrt{N} \tag{38}$$

Here the average $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$, $\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\rangle$, $\langle\rho(\vec{Q}+\vec{K}-\vec{P})d(\vec{P}-\vec{Q})\rangle\sqrt{N}$, $\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q})\rho(\vec{K}-\vec{P}-\vec{Q})\rangle\sqrt{N}$ will be calculated using the ground state wave function $|\psi_g(k)\rangle$ of the Bose-Einstein condensed 2D magnetoexcitons. It will be shown that these averages depend essentially and in some cases are proportional to the small parameter of the theory $\mathrm{v}^2(1-\mathrm{v}^2)$ related with the e–h pairs concentration. After the truncations and linearizations the multi-operator Green's functions are expressed through the one-operator Green's function $G_{1j}(\vec{P},\omega)$, with $j=1,2,3,4$, and their four equations of motion can be written in a close form introducing the self-energy parts $\Sigma_{ij}(\vec{P},\omega)$ as follows

$$\sum_{j=1}^{4}G_{1j}(\vec{P},\omega)\Sigma_{jk}(\vec{P},\omega)=C_{1k};\quad k=1,2,3,4 \tag{39}$$

There are 16 different components of the self energy part of the 4×4 matrix as follows

$$\Sigma_{11}(\vec{P},\omega) = (\hbar\omega+\bar{\mu}-E(\vec{P})+i\delta) - 4\sum_{\vec{Q}}W_{\vec{Q}}^{2}Sin^{2}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\frac{\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle}{\hbar\omega+\bar{\mu}-E(\vec{P}-\vec{Q})+i\delta} - \frac{4\eta\mathrm{v}(W_{\vec{P}-\vec{K}}N)Sin^{2}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{\hbar\omega+\bar{\mu}-E(\vec{K})+i\delta}$$

$$\Sigma_{21}(\vec{P},\omega)=\frac{4\eta e^{2i\varphi}\mathrm{v}(W_{\vec{P}-\vec{K}}N)Sin^{2}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{\hbar\omega+\bar{\mu}-E(\vec{K})+i\delta}$$

$$\Sigma_{31}(\vec{P},\omega) = -i\eta e^{i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\left(1-\frac{2(W_{\vec{P}-\vec{K}}N)}{\hbar\omega+\bar{\mu}-E(\vec{K})+i\delta}\right) + \frac{4W_{\vec{P}-\vec{K}}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{\hbar\omega+\bar{\mu}-E(\vec{K})+i\delta}\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{R}\times\vec{K}]_z l^2}{2}\right)\times\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{R})\rangle\sqrt{N} + 4\sum_{\vec{Q}}W_{\vec{Q}}W_{\vec{P}-\vec{K}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\times\frac{Sin\left(\frac{[(\vec{P}-\vec{Q})\times(\vec{P}-\vec{K})]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})\rangle\sqrt{N}}{\hbar\omega+\bar{\mu}-E(\vec{P}-\vec{Q})+i\delta}$$

$$+4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{Q}+\vec{K}-\vec{P}} - W_{\vec{P}-\vec{K}}) Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)$$
$$\times Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q})]_z l^2}{2}\right)$$
$$\times \frac{\langle\hat{\rho}(\vec{Q}+\vec{K}-\vec{P})d(\vec{P}-\vec{Q})\rangle\sqrt{N}}{\hbar\omega+\bar{\mu}-E(\vec{P}-\vec{Q})+i\delta}$$
$$\Sigma_{41}(\vec{P},\omega) = -\eta e^{i\varphi} Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) \tag{40}$$

These matrix elements form the first column of the 4×4 matrix $\|\Sigma_{ij}\|$. The second column is formed by the matrix elements $\Sigma_{j2}(\vec{P},\omega)$ with $j = 1, 2, 3, 4$ as follows

$$\Sigma_{12}(\vec{P},\omega) = 4\eta\frac{e^{-2i\varphi}\mathrm{v}(W_{\vec{K}-\vec{P}}N)Sin^2\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{\hbar\omega-\bar{\mu}+E(\vec{K})+i\delta}$$

$$\Sigma_{22}(\vec{P},\omega) = \hbar\omega-\bar{\mu}+E(2\vec{K}-\vec{P})+i\delta$$
$$-4\sum_{\vec{Q}} W^2_{\vec{Q}}\frac{Sin^2\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle}{\hbar\omega-\bar{\mu}+E(2\vec{K}-\vec{P}-\vec{Q})+i\delta}$$
$$-4\eta\mathrm{v}\frac{(W_{\vec{P}-\vec{K}}N)Sin^2\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{\hbar\omega-\bar{\mu}+E(\vec{K})+i\delta} \tag{41}$$

$$\Sigma_{32}(\vec{P},\omega)$$
$$= i\eta e^{-i\varphi} Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\left[1+\frac{2(W_{\vec{P}-\vec{K}}N)}{\hbar\omega-\bar{\mu}+E(\vec{K})+i\delta}\right]$$
$$-4\frac{W_{\vec{P}-\vec{K}}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{\hbar\omega-\bar{\mu}+E(\vec{K})+i\delta}\sum_{\vec{R}} W_{\vec{R}} Sin\left(\frac{[\vec{R}\times\vec{K}]_z l^2}{2}\right)$$
$$\times\langle d^{\dagger}(\vec{K}-\vec{R})\hat{\rho}(-\vec{R})\rangle\sqrt{N}$$
$$+4\sum_{\vec{Q}} W_{\vec{Q}}W_{\vec{K}-\vec{P}} Sin\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times Sin\left(\frac{[(2\vec{K}-\vec{P}-\vec{Q})\times(\vec{K}-\vec{P})]_z l^2}{2}\right)$$
$$\times\frac{\langle d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})\rangle\sqrt{N}}{\hbar\omega-\bar{\mu}+E(2\vec{K}-\vec{P}-\vec{Q})+i\delta}$$
$$+4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{P}-\vec{K}} - W_{\vec{P}-\vec{K}+\vec{Q}}) Sin\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times Sin\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times\frac{\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\rangle\sqrt{N}}{\hbar\omega-\bar{\mu}+E(2\vec{K}-\vec{P}-\vec{Q})+i\delta}$$

$$\Sigma_{42}(\vec{P},\omega) = \eta e^{-i\varphi} Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)$$

The third column of the 4×4 matrix $\|\Sigma_{ij}(\vec{P},\omega)\|$ consists from the self-energy parts $\Sigma_{j3}(\vec{P},\omega)$ with $j = 1, 2, 3, 4$ listed below

$$\Sigma_{13}(\vec{P},\omega) = 2i\eta e^{-i\varphi} Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)$$
$$\Sigma_{23}(\vec{P},\omega) = -2i\eta e^{i\varphi} Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)$$
$$\Sigma_{33}(\vec{P},\omega)$$
$$= (\hbar\omega+i\delta) - \frac{4}{\hbar\omega+i\delta}\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times[(W_{\vec{Q}} - W_{\vec{P}-\vec{K}})\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle + (W_{\vec{P}-\vec{K}} - W_{\vec{P}-\vec{K}-\vec{Q}})$$
$$\times\langle\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})\hat{\rho}(\vec{K}+\vec{Q}-\vec{P})\rangle]$$
$$\Sigma_{43}(\vec{P},\omega) = 0 \tag{42}$$

The fourth column is composed by the self-energy parts $\Sigma_{j4}(\vec{P},\omega)$ with $j = 1, 2, 3, 4$. They are

$$\Sigma_{14}(\vec{P},\omega) = -2\eta e^{-i\varphi} Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)$$
$$\Sigma_{24}(\vec{P},\omega) = 2\eta e^{i\varphi} Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)$$
$$\Sigma_{34}(\vec{P},\omega)$$
$$= \frac{4}{\hbar\omega+i\delta}\sum_{\vec{Q}} W_{\vec{Q}}W_{\vec{K}-\vec{P}+\vec{Q}} Sin^2\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times\langle\hat{\rho}(\vec{Q}+\vec{K}-\vec{P})\hat{D}(\vec{P}-\vec{K}-\vec{Q})\rangle \tag{43}$$
$$+\frac{4}{\hbar\omega+i\delta}\sum_{\vec{Q}} W_{\vec{Q}}W_{\vec{P}-\vec{K}} Sin^2\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)$$
$$\times[\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\rangle - \langle\hat{\rho}(\vec{Q}+\vec{K}-\vec{P})\hat{D}(\vec{P}-\vec{K}-\vec{Q})\rangle]$$
$$\Sigma_{44}(\vec{P},\omega) = (\hbar\omega+i\delta) - \frac{4}{\hbar\omega+i\delta}\sum_{\vec{Q}} W^2_{\vec{Q}}$$
$$\times Sin^2\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$$

The most of the self-energy parts $\Sigma_{ij}(\vec{P},\omega)$ represented by the formulas (40)–(43) contain the average values of the two-operator products. They were calculated using the ground state wave function $|\psi_g(k)\rangle$ (11) and have the expressions

$$\langle\psi_g(k)|\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})|\psi_g(k)\rangle = 4u^2\mathrm{v}^2 N Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)$$

$$\langle\psi_g(k)|\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})|\psi_g(k)\rangle$$
$$=4u^2\mathrm{v}^2NSin^2\left(\frac{[\vec{K}\times(\vec{P}+\vec{Q})]_z l^2}{2}\right)$$

$$\langle\psi_g(k)|\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})|\psi_g(k)\rangle = 2iu^2\mathrm{v}^2NSin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)$$

$$\langle\psi_g(k)|\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{D}(\vec{K}-\vec{P}-\vec{Q})|\psi_g(k)\rangle$$
$$=2iu^2\mathrm{v}^2NSin\left(\frac{[\vec{K}\times(\vec{P}+\vec{Q})]_z l^2}{1}\right)$$

$$\langle\psi_g(k)|d^\dagger(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})|\psi_g(k)\rangle\sqrt{N}$$
$$=2iu\mathrm{v}^3NSin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)$$

$$\langle\psi_g(k)|d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}-\vec{K})|\psi_g(k)\rangle\sqrt{N}$$
$$=-2iu\mathrm{v}^3NSin\left(\frac{[\vec{K}\times(\vec{P}-\vec{Q})]_z l^2}{2}\right)$$

$$\langle\psi_g(k)|\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})|\psi_g(k)\rangle\sqrt{N}$$
$$=-2iu\mathrm{v}^3NSin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)$$

$$\langle\psi_g(k)|\hat{\rho}(\vec{P}+\vec{Q})d(\vec{K}-\vec{P}-\vec{Q})|\psi_g(k)\rangle\sqrt{N}$$
$$=-2iu\mathrm{v}^3NSin\left(\frac{[\vec{K}\times(\vec{P}+\vec{Q})]_z l^2}{2}\right) \quad (44)$$

All these averages are extensive values proportional to N, they essentially depend on the wave vectors and on the small parameters of the types $u^2\mathrm{v}^2$ or $u\mathrm{v}^3$.

But only the averages of the type $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ are real, positive with a constant sign at any values of the wave vectors.

All another averages are pure imaginary changing their signs in dependence on their arguments leading to small absolute values of the corresponding self-energy parts. All of them will be dropped to simplify the cumbersome expressions (40)–(43).

In spite of the made approximations concerning the many operator Green's functions and the averages of the two-operator products the obtained self-energy parts remain cumbersome. But there is one possibility to radically simplify the further calculations. It is related with the collinear geometry of the experimental observation of the elementary excitations, when their propagation direction coincide or is exactly opposite with the condensate wave vector $\vec{k}$. This geometry will be discussed in the next section.

5. ELEMENTARY EXCITATIONS OF THE BOSE-EINSTEIN CONDENSED MAGNETOEXCITONS IN COLLINEAR GEOMETRY

The cumbersome dispersion equation is expressed in general form by the determinant equation

$$\det|\Sigma_{ij}(\vec{P},\omega)|=0; \quad \vec{P}=\vec{K}+\vec{q}; \quad (45)$$

It can be essentially simplified in collinear geometry, when the wave vectors $\vec{P}$ of the elementary excitations are parallel or antiparallel to the Bose-Einstein condensate wave vector $\vec{k}$. We will represent the wave vectors $\vec{P}$ in the form $\vec{P}=\vec{k}+\vec{q}$, accounting them from the condensate wave vector $\vec{k}$. The relative wave vector $\vec{q}$ will be also collinear to $\vec{k}$, In this case the projections of the wave vector products $[\vec{P}\times\vec{K}]_z$ as well as all coefficients proportional to $Sin(([\vec{P}\times\vec{K}]_z l^2)/2)$ and a half of the matrix elements $\Sigma_{ij}(\vec{P},\omega)$ in the Eq. (45) vanish. The determinant Eq. (45) disintegrates in two independent equations. One of them concerns only to optical plasmons and has a simple form

$$\Sigma_{33}(\vec{K}+\vec{q};\omega)=0; \quad [\vec{q}\times\vec{K}]_z=0 \quad (46)$$

whereas the second equation contains only the diagonal self-energy parts Σ_{11}, Σ_{22}, Σ_{44} and the quasi-average constant η

$$\Sigma_{11}(\vec{K}+\vec{q},\omega)\Sigma_{22}(\vec{K}+\vec{q},\omega)\Sigma_{44}(\vec{K}+\vec{q},\omega)$$
$$-2\eta^2(\Sigma_{11}(\vec{K}+\vec{q},\omega)+\Sigma_{22}(\vec{K}+\vec{q},\omega))=0 \quad (47)$$

It determines three interconnected branches. Two of them describe the proper collective excitations of Bose-Einstein condensed magnetoexcitons and the third branch concerns the acoustical plasmons. In spite of the collinear condition $[\vec{q}\times\vec{K}]_z=0$, the Eqs. (46) and (47) and their energy spectra $\omega(\vec{q})$ are not invariant under the inversion operation when $\vec{q}$ is substituted by $-\vec{q}$, because in the system does exist a well defined direction selected by the wave vector $\vec{k}$. By this reason the elementary excitations with wave vector $\vec{q}$ and $-\vec{q}$ have different energies.

The solutions of the dispersion Eq. (47) will be discussed in two limiting cases. One of them is the point $k=0$, where the system behaves as an ideal Bose gas and another case of considerable values of wave vectors $kl\sim 3-4$, when the Bose-Einstein condensed 2d magnetoexcitons can exist in a form of metastable dielectric liquid phase or of dielectric droplets. But in all cases the average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ and other similar expressions are determined in HFBA by the formulas (44). They are characterized by a coherence factor $Sin^2(([\vec{k}\times\vec{Q}]_z l^2)/2)$, which vanishes in the point $k=0$. All contributions to the self-energy parts proportional to square of Coulomb interaction matrix elements W_Q^2 multiplied by the averages $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ vanish also making a 2D magnetoexciton system a pure ideal gas, when the influence of the excited Landau levels is neglected. This unusual result was revealed for the first time by Lerner and Lozovik[15–17] and was confirmed by Paquet, Rice and Ueda.[19] In the case $k = 0$ because the vanishing of the averages (44) the self-energy parts become

$$\begin{gathered}\sigma_{11}(\vec{P}, \omega) = \hbar\omega + \bar{\mu} - E(P) \\ \sigma_{22}(\vec{P}, \omega) = \hbar\omega - \bar{\mu} + E(-P) \\ \sigma_{44}(\vec{P}, \omega) = \hbar\omega\end{gathered} \tag{48}$$

and the excitonic part of the dispersion relation as well the acoustic plasmon frequency look as

$$\begin{gathered}\hbar\omega_{\mathrm{ex}}(P) = \pm\sqrt{(\bar{\mu} - E(P))^2 + 4\eta^2} \\ \hbar\omega_A(P) = 0\end{gathered} \tag{49}$$

The values $\bar{\mu} = E(k)(1 - 2\mathrm{v}^2)$ and $\eta = (E(k) - \bar{\mu})\mathrm{v} = E(k)\mathrm{v}^3$ in the point $k = 0$ turn to vanish, i.e. $\bar{\mu} = \eta = E(0) = 0$, what leads to the free magnetoexciton dispersion law $\hbar\omega_{\mathrm{ex}}(P) = \pm E(P)$, and coincides with the result obtained earlier in Ref. [19]. The acoustical plasmon branch as well as the optical branch have frequencies equal to zero. The case $k \neq 0$, but $\mathrm{v} = 0$, can be obtained from the previous formula because, as earlier, the averages (44) as well as the parameter η are vanishing, whereas the chemical potential is different from zero i.e., $\bar{\mu} = E(k)$.

In this case the exciton dispersion law in collinear geometry with $P = k + qCos\alpha$ has the form

$$\begin{gathered}\hbar\omega_{\mathrm{ex}}(q) = \pm(E(k + qCos\alpha) - E(k)) \\ Cos\alpha = \pm 1,\; q > 0\end{gathered} \tag{50}$$

The both dependences are represented in Figure 1, where $x = ql$ was introduced.

The case of $k \neq 0$ with filling factor $v = \mathrm{v}^2 < 1$ represents interest because in this region of parameters a metastable dielectric liquid phase does exist. It is formed by the Bose-Einstein condensed magnteoexcitons with $kl \sim 3$–4 and with different from zero motional dipole moments $\vec{\rho} = [\vec{k} \times \vec{z}]l^2$. This state was revealed in Ref. [20] considering the system of electrons and holes on their lowest Landau levels, without addressing to excited Landau levels (ELLs), but taking into account coherent excited states, when one e–h pair exits from the condensate leaving all another pairs in their coherent pairing state.

The correlation energy was calculated beyond the Hartree-Fock-Bogoliubov approximation (HFBA) in the frame of Keldysh-Kozlov-Kopaev method using the Nozieres Comte approach.[20, 39]

The Bose-Einstein condensed magnetoexcitons moving as a whole with wave vector $\vec{k}$ and with parallel motional dipole moments $\vec{\rho}$ have a significant polarizability which gives rise to attractive interaction between them and lower on the energy scale the values of the chemical potential and of the mean energy per one e–h pair. But this lowering is not monotonous and at some value of the filling factor $\mathrm{v}_{\mathrm{m}}^2$ the relative minim a on the corresponding curves appear with positive compressibilities in their vicinity. The relative minimum on the chemical potential curve depends essentially on the damping of magnetoexciton level. It was investigated in the Ref. [21] and is represented in Figure 2.

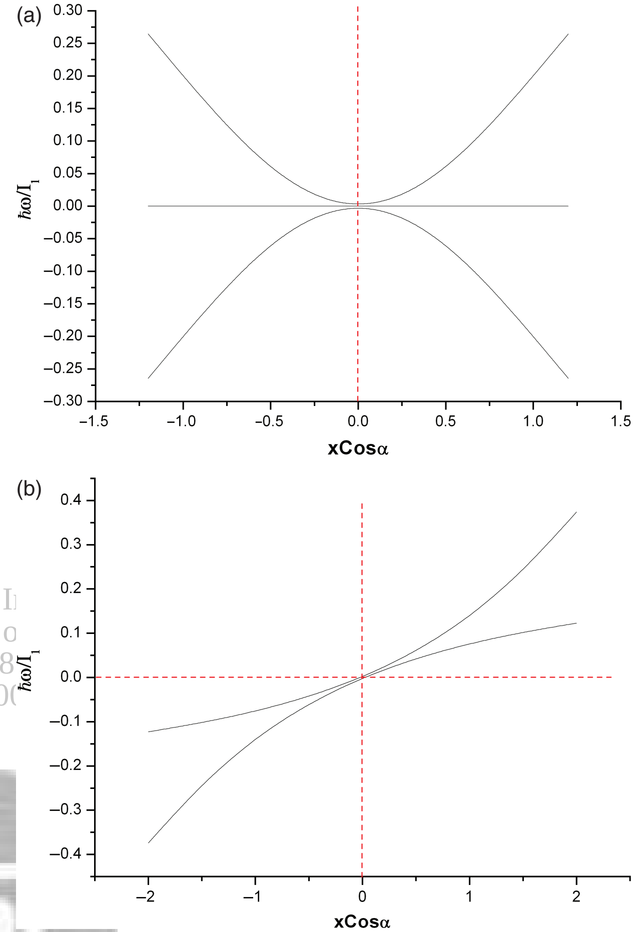

Fig. 1. The energy spectrum of elementary excitations of magnetoexcitons and acoustical plasmons in the case when concentrations corrections haven't been taken into account. (a) The wave vector of BEC magnetoexcitons equal to 0. (b) The wave vector k is different from zero, but the filling factor equals to zero.

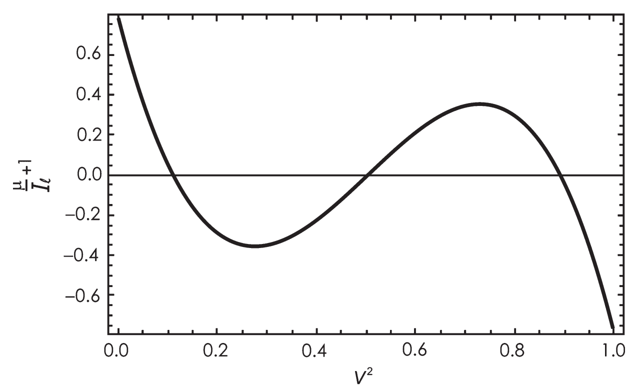

Fig. 2. The relative minimum on the chemical potential dependence on the filling factor.

If the average filling factor v^2 is less than v_m^2 the dielectric liquid phase will exist in the form of droplets with optimal concentration inside them $n_{ex} = v_m^2/2\pi l_0^2$ corresponding to filling factor v_m^2.

The collective elementary excitations are calculated in the conditions $kl \sim 3$–4 and $v^2 \approx v_m^2$, when the ground state of the magnetoexcitons is similar with the metastable dielectric liquid phase.

Even in collinear geometry the diagonal self-energy parts $\Sigma_{ii}(\vec{K}+\vec{q},\omega)$ with $i = 1,2,3,4$ and $kl = 3,6$ can not be calculated analytically at arbitrary values of the relative wave vector $\vec{q}$. By this reason we will obtain the analytical expressions in the case $kl \approx 3,6$ and $ql \le 1 < kl$ using a series expansions on the small values $ql < 1$ as compared with $kl \approx 3,6$.

The self-energy parts $\Sigma_{11}(\vec{k}+\vec{q},\omega)$ and $\Sigma_{22}(\vec{k}+\vec{q},\omega)$ in collinear geometry have the forms

$$\begin{aligned}
&\Sigma_{11}(\vec{k}+\vec{q},\omega)\\
&\quad= (\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})+i\delta)\\
&\qquad -4\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{k}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad \times\frac{\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle}{\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}-\vec{Q})+i\delta}\\
&\quad= \sigma_{11}(\vec{k}+\vec{q},\omega)+i\Gamma_{11}(\vec{k}+\vec{q},\omega)\\
&\Sigma_{22}(\vec{k}+\vec{q},\omega)\\
&\quad= (\hbar\omega-\bar{\mu}+E(\vec{k}-\vec{q})+i\delta)\\
&\qquad -4\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{k}-\vec{q})\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad \times\frac{\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle}{\hbar\omega-\bar{\mu}+E(\vec{k}-\vec{q}-\vec{Q})+i\delta}\\
&\quad= \sigma_{22}(\vec{k}+\vec{q},\omega)+i\Gamma_{22}(\vec{k}+\vec{q},\omega)
\end{aligned}\tag{51}$$

They were devided in real and imaginary parts and obey to the equalities

$$\begin{aligned}
\Sigma_{22}(\vec{k}+\vec{q},\omega) &= -\Sigma_{11}^*(\vec{k}-\vec{q},-\omega)\\
\sigma_{22}(\vec{k}+\vec{q},\omega) &= -\sigma_{11}(\vec{k}-\vec{q},-\omega)\\
\Gamma_{22}(\vec{k}+\vec{q},\omega) &= \Gamma_{11}(\vec{k}-\vec{q},-\omega)
\end{aligned}\tag{52}$$

The real and imaginary parts σ_{11} and Γ_{11} are

$$\begin{aligned}
&\sigma_{11}(\vec{k}+\vec{q},\omega)\\
&\quad= \hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})-4\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{k}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad\times\frac{Pf\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle}{\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}-\vec{Q})}
\end{aligned}\tag{53}$$

$$\begin{aligned}
&\Gamma_{11}(\vec{k}+\vec{q},\omega)\\
&\quad= 4\pi\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{k}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad\times\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle\delta(\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}-\vec{Q}))
\end{aligned}$$

They will be expanded in series expansions on the parameter $ql < 1$, which is small in comparisons with considerable value $kl \approx 3,6$. At the same time we will expand the denominator $\hbar\omega+\bar{\mu}-E(\vec{K}+\vec{q}-\vec{Q})$ in the formula (53) taking into account that the most probable value of Ql is unity ($\overline{Ql} \approx 1$) and $\overline{Ql}$ is less than the elementary excitation wave vector $|\vec{k}+\vec{q}|l$.

In this approximation, when we are concerning with the elementary excitations of the metastable dielectric liquid phase with $kl \approx 3,6$; $ql < 1$; $\overline{Ql} \approx 1$ we can write

$$\begin{aligned}
&\frac{1}{\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}-\vec{Q})}\\
&\quad\approx\frac{1}{\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})}\\
&\qquad+\frac{1}{(\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}))^2}\cdot\frac{\vec{Q}\partial E(\vec{P})}{\partial\vec{P}}\bigg|_{\vec{P}=\vec{k}+\vec{q}}\\
&\qquad+\frac{1}{2}\frac{1}{(\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}))^2}\cdot\frac{\vec{Q}^2\partial^2 E(\vec{P})}{\partial P^2}\bigg|_{P=|\vec{k}+\vec{q}|}
\end{aligned}\tag{54}$$

Substituting (54) into the real part (53), the middle term can be dropped due to the even dependence on $\vec{Q}$ of the remaining part of the under integral expression (53). The real part $\sigma_{11}(\vec{K}+\vec{q},\omega)$ can be represented in the form

$$\begin{aligned}
&\sigma_{11}(\vec{K}+\vec{q},\omega)\\
&\quad= \hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})-\frac{4}{\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})}\\
&\qquad\times\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{k}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle\\
&\qquad-\frac{2\left(\frac{\partial^2 E(P)}{\partial P^2}\right)\Big|_{P=|\vec{k}+\vec{q}|}}{(\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}))^2}\sum_{\vec{Q}} W_{\vec{Q}}^2 Q^2\\
&\qquad\times Sin^2\left(\frac{[(\vec{k}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle
\end{aligned}\tag{55}$$

The imaginary part will become

$$\begin{aligned}
&\Gamma_{11}(\vec{K}+\vec{q},\omega)\\
&\quad=4\pi\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{K}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)
\end{aligned}$$

$$\times \langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle\delta\left(\hbar\omega+\bar{\mu}-E(\vec{K}+\vec{q})+\frac{\vec{Q}\partial E(\vec{P})}{\partial\vec{P}}\bigg|_{\vec{P}=\vec{k}+\vec{q}} -\frac{1}{2}\frac{\vec{Q}^2\partial^2 E(\vec{P})}{\partial\vec{P}^2}\bigg|_{P=|\vec{k}+\vec{q}|}\right) \tag{56}$$

The average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ and the chemical potential $\bar{\mu}$ have the expressions

$$\bar{\mu}=E(k)(1-2\mathrm{v}^2)$$

$$\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle = 4u^2\mathrm{v}^2 NSin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \tag{57}$$

$$I_{\text{ex}}(k)=I_l-E(k);\quad \bar{\mu}=\mu+I_l;\quad I_l=\frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}}$$

$$u^2=1-\mathrm{v}^2;\quad E(k)=2\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)$$

$$\eta=(E(k)-\bar{\mu})\mathrm{v};\quad v=\mathrm{v}^2$$

Now the series expansions for the self-energy parts $\sigma_{11}(\vec{K}+\vec{q},\omega)$ and $\Gamma_{11}(\vec{K}+\vec{q},\omega)$ will be represented, introducing in explicit form the average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$. They are

$$\begin{aligned}
&\sigma_{11}(\vec{K}+\vec{q},\omega)\\
&\approx \hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})-\frac{16u^2\mathrm{v}^2}{\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q})}\\
&\quad\times\left\{\sum_{\vec{Q}} W^2_{\vec{Q}} NSin^4\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\right.\\
&\quad+\sum_{\vec{Q}} W^2_{\vec{Q}} NSin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad+\sum_{\vec{Q}} W^2_{\vec{Q}} NSin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\left.\times Cos\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)^2\right\}\\
&\quad-\frac{8u^2\mathrm{v}^2\frac{\partial^2 E(P)}{\partial P^2}\big|_{P=|\vec{k}+\vec{q}|}}{(\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}))^2}\\
&\quad\times\left\{\sum_{\vec{Q}} W^2_{\vec{Q}} N\vec{Q}^2 Sin^4\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\right.\\
&\quad+\sum_{\vec{Q}} W^2_{\vec{Q}} N\vec{Q}^2 Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad+\sum_{\vec{Q}} W^2_{\vec{Q}} N\vec{Q}^2 Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\left.\times Cos\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)^2\right\}
\end{aligned} \tag{58}$$

and

$$\begin{aligned}
&\Gamma_{11}(\vec{K}+\vec{q},\omega)\\
&=16\pi u^2\mathrm{v}^2\sum_{\vec{Q}} W^2_{\vec{Q}} NSin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\times Sin^2\left(\frac{[(\vec{K}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\delta(\hbar\omega+\bar{\mu}-E(\vec{k}+\vec{q}-\vec{Q}))
\end{aligned} \tag{59}$$

where

$$\begin{aligned}
&Sin^2\left(\frac{[(\vec{K}+\vec{q})\times\vec{Q}]_z l^2}{2}\right)\\
&\quad\cong Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)+Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad+Cos\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)^2
\end{aligned} \tag{60}$$

The coefficients in expression (58) determining the real self-energy part $\sigma_{11}(\vec{K}+\vec{q},\omega)$ were calculated analytically exactly. The dimensionless wave vectors of the elementary excitations $Pl=z$, of the condensate $kl=y\approx 3,6$ and of the relative wave vector $ql=x<1$, were introduced. In the collinear geometry and $x<y$, we can write

$$z=|y+xCos\alpha|,\quad \text{where } \alpha=0,\pi,\ Cos\alpha=\pm 1 \tag{61}$$

The first coefficients $C(y)$ is determined as[44]

$$C(y)=\sum_{\vec{Q}} W^2_{\vec{Q}} NSin^4\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)=I_l^2 C_1(y)$$

$$C_1(y)=\frac{1}{8\pi}\left\{E_i(-y^2)-4E_i\left(-\frac{y^2}{4}\right)+3\ln(y^2)+3C-8\ln 2\right\}$$

$$=\frac{1}{8\pi}\left\{\sum_{k=1}^{\infty}\frac{(-y^2)^k}{k(k!)}-4\sum_{k=1}^{\infty}\frac{(-\frac{y^2}{4})^k}{k(k!)}\right\}$$

$$C=0.577216-\text{Euler constant} \tag{62}$$

Here $E_i(y)$ is the integral exponential function.[44] In the limit $y\to 0$ the figured bracket is proportional to y^4, as one can expect looking at the starting expression. This integral in the paper[20] was calculated approximately and its limiting value at $y\to 0$ differs by $\sqrt{\pi}$ from its exact value.

The second coefficient $L(x, y)$ depend on $xCos\alpha$ and they will be represented in the forms

$$L(x, y) = \sum_{\vec{Q}} W_{\vec{Q}}^2 N Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \times Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right) = I_l^2 x Cos\alpha L_1(y);$$

$$L_1(y) = \frac{1}{4\pi} y\left[\exp\left(-\frac{y^2}{4}\right)_i F_1\left(1; 2; \frac{y^2}{4}\right) - \exp(-y^2)_i F_1(1; 2; y^2)\right] \quad (63)$$

The limiting values of $L(x, y)$ are proportional to y^3 correspondingly at $y \to 0$. The third $P(x, y)$ coefficient is proportional to x^2:

$$P(x, y) = \sum_{\vec{Q}} W_{\vec{Q}}^2 N Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \times Cos\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)^2 = I_l^2 x^2 P_1(y)$$

$$P_1(y) = \frac{1}{16\pi}\left\{-\frac{1}{2} + \exp\left(-\frac{y^2}{4}\right) - \frac{1}{2}\exp(-y^2) + \frac{y^2}{4}\left[\exp(-y^2)_i F_1(2; 3; y^2) - \frac{1}{2}\exp\left(-\frac{y^2}{4}\right)_i F_1\left(2; 3; \frac{y^2}{4}\right)\right]\right\} \quad (64)$$

The limiting value of $P(x, y)$ is proportional to y^2 at $y \to 0$. The fourth $F(y)$ coefficient is

$$F(y) = \sum_{\vec{Q}} W_{\vec{Q}}^2 N Q^2 Sin^4\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) = \frac{I_l^2}{l^2} F_1(y)$$

$$F_1(y) = \frac{1}{\pi}\left[\frac{3}{8} - \frac{1}{2}\exp\left(-\frac{y^2}{4}\right) + \frac{1}{8}\exp(-y^2)\right] \quad (65)$$

The last coefficient $S(x, y)$ is proportional to x^2, and look as follows

$$S(x, y) = \sum_{\vec{Q}} W_{\vec{Q}}^2 N Q^2 Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \times Cos\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{1}\right)\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)^2 = \frac{I_l^2}{l^2} x^2 S_1(y)$$

$$S_1(y) = \frac{1}{16\pi}\left\{-\frac{1}{2} + \exp\left(-\frac{y^2}{4}\right)_i F_1\left(-1; 1; \frac{y^2}{4}\right) - \frac{1}{2}\exp(-y^2)_i F_1(-1; 1; y^2) - \frac{y^2}{4}\exp\left(-\frac{y^2}{4}\right) + \frac{y^2}{2}\exp(-y^2)\right\} \quad (66)$$

Now the coefficients (62)–(66) will be substituted into the expression $\sigma_{11}(\vec{k}+\vec{q}, \omega)$ and the dimensionless values will be introduced

$$\frac{\hbar\omega}{I_l} = \tilde{\omega}, \quad \frac{\bar{\mu}}{I_l} = \tilde{\mu}, \quad \frac{E(p)}{I_l} = \tilde{E}(p) \quad \frac{\sigma_{11}}{I_l} = \tilde{\sigma}_{11}$$
$$\frac{\Gamma_{11}}{I_l} = \tilde{\Gamma}_{11}, \quad \frac{I_{ex}(k)}{I_l} = \tilde{I}_{ex}(k), \quad Pl = z \quad (67)$$
$$kl = y, \quad ql = x, \quad Cos\alpha = \pm 1, \quad \alpha = 0, \pi$$

They transform $\tilde{\sigma}_{11}(x, y, \tilde{\omega})$ into the form

$$\tilde{\sigma}_{11}(xCos\alpha, y, \tilde{\omega}) = u_{11}(xCos\alpha, y, \tilde{\omega}) + \mathrm{v}_{11}(xCos\alpha, y, \tilde{\omega})xCos\alpha + x^2 W_{11}(xCos\alpha, y, \tilde{\omega}) \quad (68)$$

where

$$u_{11}(xCos\alpha, y, \tilde{\omega}) = \tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha) - \frac{16u^2\mathrm{v}^2 C_1(y)}{\tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha)} - \frac{8u^2\mathrm{v}^2 F_1(y)\frac{\partial^2\tilde{E}(z)}{\partial z^2}\Big|_{z=|y+xCos\alpha|}}{(\tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha))^2}$$

$$\mathrm{v}_{11}(xCos\alpha, y, \tilde{\omega}) = -\frac{16u^2\mathrm{v}^2 L_1(y)}{\tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha)} - \frac{8u^2\mathrm{v}^2 N_1(y)\frac{\partial^2\tilde{E}(z)}{\partial z^2}\Big|_{z=|y+xCos\alpha|}}{(\tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha))^2} \quad (69)$$

$$W_{11}(x, y, \tilde{\omega}) = -\frac{16u^2\mathrm{v}^2 P_1(y)}{\tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha)} - \frac{8u^2\mathrm{v}^2 S_1(y)\frac{\partial^2\tilde{E}(z)}{\partial z^2}\Big|_{z=|y+xCos\alpha|}}{(\tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha))^2}$$

The dispersion relation $\tilde{E}(y)$ for the magnetoexciton will be approximated as

$$\tilde{E}(y) = \frac{y^2}{4+y^2}; \quad \tilde{I}_{ex}(y) = \frac{4}{4+y^2}; \quad y = kl$$
$$\frac{\partial\tilde{E}(y)}{\partial y} = \frac{8y}{(4+y^2)^2}; \quad \frac{\partial^2\tilde{E}(y)}{\partial y^2} = \frac{8(4-3y^2)}{(4+y^2)^3} \quad (70)$$

It reflects the quadratic dependence of $\tilde{E}(y)$ in the region of small y as well as the approaching of $\tilde{E}(y)$ to unity when y tends to infinity. The dimensionless chemical potential obtained in the HFBA

$$\begin{gathered}\tilde{\mu}(y)=\tilde{E}(y)(1-2\mathrm{v}^2); \quad \eta=(E(k)-\bar{\mu})\mathrm{v} \\ \eta/I_l=\tilde{\eta}=(\tilde{E}(k)-\tilde{\mu})\mathrm{v}\end{gathered} \tag{71}$$

Expanding the real self-energy part $\sigma_{22}(\vec{k}+\vec{q},\omega)$ in the series expansions in a similar way as it was made with the self-energy part $\sigma_{11}(\vec{k}+\vec{q},\omega)$ we will obtain the expression

$$\begin{aligned}&\sigma_{22}(xCos\alpha, y, \tilde{\omega}) \\ &\quad = u_{22}(xCos\alpha, y, \tilde{\omega})+xCos\alpha \mathrm{v}_{22}(xCos\alpha, y, \tilde{\omega}) \\ &\qquad + x^2\mathrm{w}_{22}(xCos\alpha, y, \tilde{\omega}) \\ &\quad = -\sigma_{11}(-xCos\alpha, y, -\tilde{\omega})\end{aligned} \tag{72}$$

Here the notations $x=ql$, $y=kl$, $\alpha=0,\pi$, $Cos\alpha=\pm 1$, $\tilde{\omega}=(\hbar\omega/I_l)$ were used. The coefficient u_{22}, v_{22} and w_{22} are

$$\begin{aligned}&u_{22}(xCos\alpha, y, \tilde{\omega}) \\ &\quad = \tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha)-\frac{16u^2\mathrm{v}^2C_1(y)}{(\tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha))} \\ &\qquad + \frac{8u^2\mathrm{v}^2F_1(y)\frac{\partial^2\tilde{E}(z)}{\partial z^2}\big|_{z=|y-xCos\alpha|}}{(\tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha))^2} \\ &\quad = -u_{11}(-xCos\alpha, y, -\tilde{\omega})\end{aligned}$$

$$\begin{aligned}&\mathrm{v}_{22}(xCos\alpha, y, \tilde{\omega}) \\ &\quad = \frac{16u^2\mathrm{v}^2L_1(y)}{(\tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha))} \\ &\qquad - \frac{8u^2\mathrm{v}^2N_1(y)\frac{\partial^2\tilde{E}(z)}{\partial z^2}\big|_{z=|y-xCos\alpha|}}{(\tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha))^2} \\ &\quad = \mathrm{v}_{11}(-xCos\alpha, y, -\tilde{\omega})\end{aligned} \tag{73}$$

$$\begin{aligned}\mathrm{w}_{22}(xCos\alpha, y, \tilde{\omega}) &= -\frac{16u^2\mathrm{v}^2P_1(y)}{(\tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha))} \\ &\quad + \frac{8u^2\mathrm{v}^2S_1(y)\frac{\partial^2\tilde{E}(z)}{\partial z^2}\big|_{z=|y-xCos\alpha|}}{(\tilde{\omega}-\tilde{\mu}+\tilde{E}(y-xCos\alpha))^2} \\ &= -\mathrm{w}_{11}(-xCos\alpha, y, -\tilde{\omega})\end{aligned}$$

The imaginary part $\Gamma_{22}(\vec{k}+\vec{q},\omega)$ equals

$$\begin{aligned}\Gamma_{22}(\vec{k}+\vec{q},\omega) &= 4\pi\sum_{\vec{Q}}W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{k}-\vec{q})\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle\delta(\hbar\omega-\bar{\mu}+E(\vec{k}-\vec{q}-\vec{Q}))\end{aligned} \tag{74}$$

Now the remaining two diagonal self-energy parts Σ_{33} and Σ_{44} will be considered. Their imaginary components equal to zero.

The starting expression

$$\begin{aligned}\Sigma_{44}(\vec{k}+\vec{q},\omega) &= (\hbar\omega+i\delta)-\frac{4}{\hbar\omega+i\delta} \\ &\quad \times\sum_{\vec{Q}}W_{\vec{Q}}^2 Sin^2\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle\end{aligned} \tag{75}$$

and the formula (56) for the average $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ lead to real part of Σ_{44} and imaginary parts of Σ_{22}

$$\begin{aligned}\sigma_{44}(\vec{k}+\vec{q},\omega) &= \hbar\omega-\frac{16u^2\mathrm{v}^2}{\hbar\omega}\sum_{\vec{Q}}W_{\vec{Q}}^2 N Sin^2\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times Sin^2\left(\frac{[\vec{k}\times\vec{Q}]_z l^2}{2}\right) \\ \Gamma_{22}(\vec{k}+\vec{q},\omega) &= 16\pi u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}^2 N Sin^2\left(\frac{[\vec{k}\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times Sin^2\left(\frac{[(\vec{k}-\vec{q})\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times\delta(\hbar\omega-\bar{\mu}+E(\vec{k}-\vec{q}-\vec{Q})) \\ &= \Gamma_{11}(\vec{k}-\vec{q},-\omega)\end{aligned} \tag{76}$$

We are interested in the range of relative wave vectors $ql<1$. Taking into account that the most probable value of the variable Q lies in the vicinity of $Ql\cong 1$ we can substitute $Sin^2(([\vec{q}\times\vec{Q}]_z l^2)/2)$ by $(([\vec{q}\times\vec{Q}]_z l^2)/2)^2$. This approximation leads to the final expression of σ_{44} dependence on the dimensionless frequency $\tilde{\omega}$ and wave vectors $x=ql$ and $y=kl$

$$\sigma_{44}(x,y,\tilde{\omega})=\tilde{\omega}-\frac{x^2}{\tilde{\omega}}\left[u^2\mathrm{v}^2\frac{2}{\pi}A_1(y)\right] \tag{77}$$

where the coefficient $A_1(y)$ is

$$A_1(y)=\frac{1}{2}-\frac{1}{2}e^{-y^2/4}+\frac{y^2}{16}e^{-y^2/4}{}_1F_1\left(1;3;\frac{y^2}{4}\right) \tag{78}$$

The last diagonal self-energy part in our enumeration $\Sigma_{33}(\vec{k}+\vec{q},\omega)$ has the starting form

$$\begin{aligned}&\Sigma_{33}(\vec{k}+\vec{q},\omega) \\ &\quad = (\hbar\omega+i\delta)-\frac{4}{\hbar\omega+i\delta}\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right) \\ &\qquad \times\{W_{\vec{Q}}\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle-W_{\vec{Q}-\vec{q}}\langle\hat{\rho}(\vec{q}-\vec{Q})\hat{\rho}(\vec{Q}-\vec{q})\rangle \\ &\qquad + W_{\vec{q}}[\langle\hat{\rho}(\vec{q}-\vec{Q})\hat{\rho}(\vec{Q}-\vec{q})\rangle-\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle]\}\end{aligned} \tag{79}$$

It was selected separately from another self-energy parts and this separation was possible only in the collinear geometry. This geometry permits to continue the simplification, because the averages $\langle\hat{\rho}(\vec{q}-\vec{Q})\hat{\rho}(\vec{Q}-\vec{q})\rangle$ and $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ coincide, when $[\vec{q}\times\vec{k}]_z = 0$ and the last square bracket term vanishes.

The simplified expression for $\sigma_{33}(\vec{k}+\vec{q}, \omega)$ looks as

$$\sigma_{33}(x, y, \omega) = \hbar\omega - \frac{4}{\hbar\omega}\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{Q}} - W_{\vec{Q}-\vec{q}}) \times Sin^2\left(\frac{[\vec{q}\times\vec{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle \quad (80)$$

The self-energy part (79) in the case of Bose-Einstein condensed magnetoexcitons depends on the average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ (45). In its turn $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ essentially depends on the condensate wave vector $\vec{k}$ and vanish in the point $k = 0$. As was shown in Ref. [46] in the case of electron–hole liquid (EHL) the self-energy part $\sigma_{33}(q, \omega)$ describing the intra- lowest-Landau level excitations has the similar form, but with the average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle = 2\text{v}^2(1-\text{v}^2)N$.

Below the numerical calculations on the base of the above derived analytical formulas will be presented.

Up till now we have discussed the energy spectrum of a Bose-Einstein condensed magnetoexcitons in pure ideal conditions which take place in the case $k = 0$, when the interactions in the electron–hole system are reciprocally compensated at arbitrary values of the filling factor $\text{v}^2 \neq 0$, as well as in the case $k \neq 0$, when the nonlinearity is completely neglected putting $v = 0$. In the last case taking into account the nonlinearity $\text{v}^2 \neq 0$ we can observe its unusual influence on the earlier discussed energy spectrum leading to its qualitative new and principal changes. They are different from the simple additions of the concentration corrections to the exciton branches of spectrum as one could expect on the base of a simple perturbation theory. Instead of it the influence of the concentration terms proportional to $u^2\text{v}^2$ entering into the compositions of the self-energy parts σ_{11}, σ_{22} and σ_{44} happens to be much more important. The self-energy parts contain the different linear on $\tilde{\omega}$ expressions of the type $L_i(\tilde{\omega}) = \tilde{\omega} + \tilde{\mu} - \tilde{E}(y + xCos\alpha)$ which appear in the forms $A_i/L_i(\tilde{\omega})$ and determine the concentration corrections. For simplicity we will demonstrate their influence taking into account only the denominators in the first power. The self-energy parts σ_{11} and σ_{22} contain also such denominators in power two of the forms $B_i/(L_i(\tilde{\omega}))^2$, but these terms for simplicity were neglected in the numerical calculations. The presence of the unknown frequency $\tilde{\omega}$ in the denominators side by side with another term in numerators leads to the increasing of the order of the dispersion equation and of the number of the energy spectrum branches. In our concrete case the order of dispersion equation is doubled and instead of three branches of the energy spectrum we are dealing with six branches. Two of

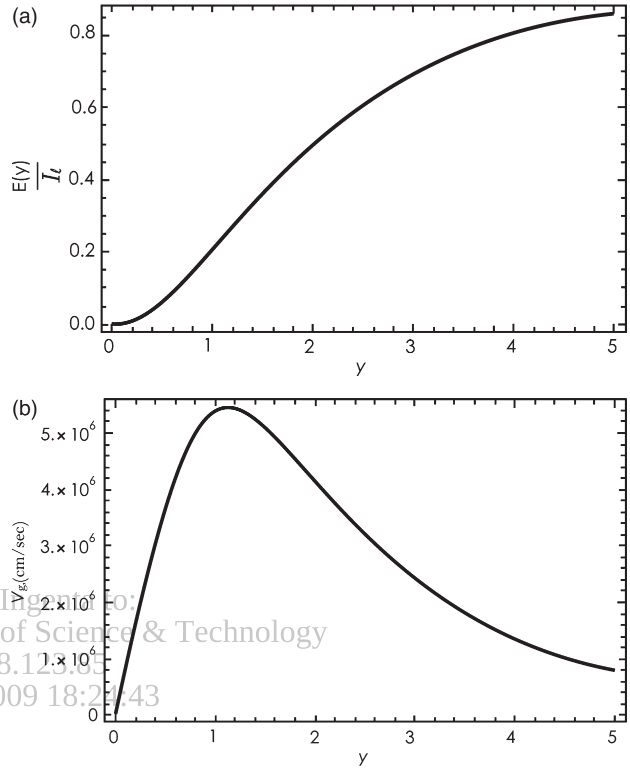

Fig. 3. (a) The dispersion law of the magnetoexciton. (b) The group velocity $V_g(k)$ of the magnetoexciton.

them are acoustical plasmon branches with energies proportional to the perturbation theory parameter $\text{v}^2(1-\text{v}^2)$ and with different $\pm$ signs. It was natural to expect the appearance of these two branches of acoustical plasmon spectrum and the same takes place with the optical plasmon spectrum. Unusual behavior happens with the exciton energy and quasienergy branches which become doubled undergoing each of them a bifurcation. The new branches

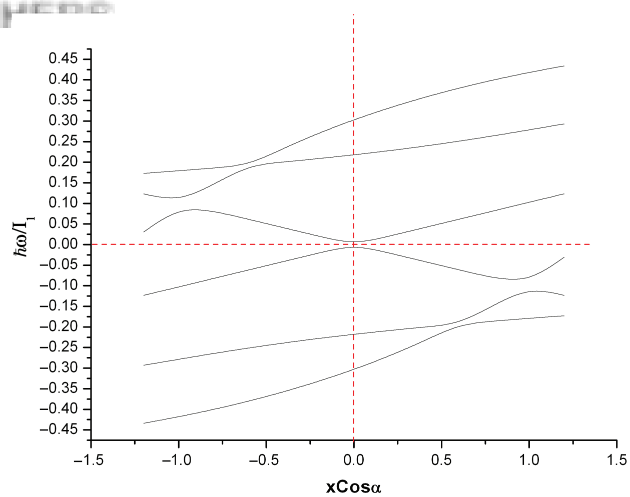

Fig. 4. The energy spectrum of elementary excitations of magnetoexcitons and acoustical plasmons in the case when filling factor of the lowest Landau levels equals to $\text{v}^2 = 0.028$. The dimensionless wave vector of the Bose-Einstein condensed magnetoexcitons equals to 3, 6.

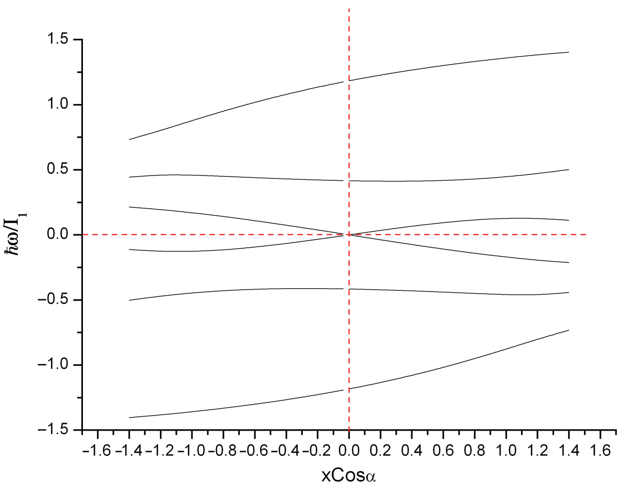

Fig. 5. The energy spectrum of elementary excitations of magnetoexcitons and acoustical plasmons in the case when filling factor of the lowest Landau levels equals to $v^2 = 0.28$. The dimensionless the wave vector of the Bose-Einstein condensed magnetoexcitons equals to 3, 6.

have the form of the previous exciton branch plus or minus one additional of amount approximately equal to the energy of the acoustical plasmon with wave vector different from the wave vector of the exciton elementary excitation by the condensate wave vector k. The same change takes place with the quasienergy exciton branch. The neglected denominator in power two could create exciton branch with two acoustical plasmons. The Bose-Einstein condensation with $k \neq 0$ means that the e–h system is moving as regards the laboratory reference frame with a velocity equal to the group velocity V_g of the magnetoexcitons, reflected in the Figure 3. It means that the terms $\hbar\vec{V}_g\vec{q}$ will appear in the dispersion relations for all three branches. To create the exciton-type collective elementary excitations when the ground state of the system is a dielectric liquid phase with negative values of the chemical potential μ it is necessary to eliberate an exciton from the liquid communicating it an amount of energy at least equal to $|\mu|$. This values $|\mu|$ are equal to $0.31I_l$ and $0.69I_l$ at the filling factors v^2 equal to 0.028 and 0.28 correspondingly. Because the concentration corrections to the energy spectrum in our case appear in the form of acoustical plasmon energy $\hbar\omega_{\mathrm{AP}}$ two exciton branches have approximately the energies $|\mu| \pm \hbar\omega_{\mathrm{AP}}$. The exciton and plasmon quasienergy branches can be obtained from the exciton and plasmon energy branches by two successive reflections as regards two coordinate axes. These properties can be observed on the Figures 4 and 5.

6. CONCLUSION

The collective elementary excitations of a system of Bose-Einstein condensed magnetoexcitons interacting with electron–hole plasma in a strong perpendicular magnetic field were studied. The breaking of the gauge symmetry was introduced into the Hamiltonian following the Bogoliubov's theory of quasiaverages. The equations of motion for integral two-particle operators describing the creation and annihilation of magnetoexcitons as well as the electron–hole plasma density fluctuations were derived. The two-particle operators were used to construct four types of Green's functions. Two of them are normal and anormal exciton Green's functions whereas another two describe the acoustical and optical plasmons. The Green's functions obey to four equations of motion, which contain nonlinearity and higher order Green's functions, for which another more complicate equations of motion were obtained. The chains of equations of motion containing the six-particle Green's functions were truncated expressing approximately the six-particle Green's functions through the two-particle Green's functions multiplied by the average values of the four-particle operators. This deconnection procedure permits to obtain an enclosed set of four Dyson equations with self-energy parts Σ_{ij} with $i, j = 1, 2, 3, 4$ forming a 4×4 matrix. Its determinant gives rise to four order dispersion equation, the elements of which are the self-energy parts. In collinear geometry of observation when the elementary excitation wave vectors are collinear with the condensate wave vector the dispersion equation desintegrates in two independent equations. One of them contains only the self-energy part of the optical plasmons, whereas the second third order dispersion equation contains the diagonal self-energy parts of other three components. In their compositions there are denominators containing the unknown frequency what doubled the order of the dispersion equation transforming it from three to six order. Six branches of the energy spectrum describe two exciton-plasmon energy branches, two exciton-plasmon quasienergy branches and two of them with ± signs belong to acoustical plasmon branches.

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