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Collective states of two-dimensional electron–hole system under the influence of Rashba spin–orbit coupling

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This study is concerned with a two-dimensional (2D) electron–hole (*e–h*) system in a strong perpendicular magnetic field with special attention devoted to the Rashba spin–orbit coupling (RSOC). The influence of this interaction on the chemical potential of the Bose–Einstein condensed magnetoexcitons and on the ground-state energy of the metallic-type electron–hole liquid (EHL) is investigated in the Hartree–Fock approximation (HFA). The magnetoexciton ground-state energy, and the energy of the single-particle elementary excitations were obtained. We demonstrated that chemical potential is a monotonic function versus the value of the filling factor with negative compressibility, which leads to instability of the Bose–Einstein condensate of magnetoexcitons. The energy per one *e–h* pair inside the electron–hole droplets (EHD) is found to be situated on the energy scale lower than the value of the chemical potential of the Bose–Einstein condensed magnetoexcitons with wave vector $\boldsymbol{k} = 0$ calculated in the HFA.

1 Introduction Properties of the symmetric 2D *e–h* system (*i.e.*, $h = 0$), with equal concentrations for both components, with coincident matrix elements of Coulomb electron–electron, hole–hole, and *e–h* interactions in a strong perpendicular magnetic field also attracted a great attention during the last two decades [1–8]. A hidden symmetry and the multiplicative states were discussed in many papers [5, 9, 10]. The collective states such as the Bose–Einstein condensation (BEC) of 2D magnetoexcitons and the formation of metallic-type electron–hole liquid were investigated in Refs. [1–8]. The search for Bose–Einstein condensates has become a milestone in condensed-matter physics [11]. The remarkable properties of superfluids and superconductors are intimately related to the existence of a bosonic condensate of composite particles consisting of an even number of fermions. In highly excited semiconductors the role of such composite bosons is taken on by excitons, which are bound states of electrons and holes. Furthermore, the excitonic system has been viewed as a keystone system for exploration of the BEC phenomena, since it allows control of particle densities and interactions in situ. Promising candidates for experimental realization of such a system are semiconductor quantum wells (QWs) [12], which have a number of advantages compared to the bulk systems.

In Ref. [13] Fertig investigated the energy spectrum of a bilayer electron system in a strong perpendicular magnetic field and introduced the concept of the interlayer phase coherence of the electron states in two adjacent layers, which leads to the model of quantum Hall excitons (QHExs) under the condition of their BEC. Unexpectedly a strong evidence of exciton BEC was ultimately found in such surprising system as a double-layer 2D electron system at a high magnetic field [14]. In the regime of the quantum Hall effect the excitons consist of electrons in the lowest Landau level (LLL) of the conduction band of one layer being bound to the holes that appear in the LLL of the conduction band in another layer. The formation of such unusual holes is due to the possibility to consider the half-filled LLL by electrons of the conduction band, for example, of the first layer as being completely filled by electrons with filling factor $\nu = 1$ and simultaneously being half-filled by holes in the same conduction band. The full-filling electrons of the first layer are considered as being compensated by the impurity-doped adjacent layer and the theoretical model takes into account only the holes in the first layer and the electrons in the second

layer. Both components belong to the LLLs of the same conduction band and are characterized by a half-filling factor for each of them. This new type of excitons named QHExs and introduced by Fertig [13] exhibits several dramatic electrical transport properties revealed in Refs. [15–17]. As was mentioned in Ref. [14] the BEC of the QHExs reflects the spontaneously broken $U(1)$ symmetry in which the electrons are no longer confined to one layer or to the other, but instead they reside in a coherent linear combination of the two layers. This interlayer phase coherence develops only when the effective interlayer separation d/l is less than a critical value $(d/l)_c$, where l is the magnetic length. At large d/l the bilayer system behaves qualitatively like the independent 2D electron systems. Following [18] this new state can be distinguished as a Fermi-liquid state of composite fermions. It is unique because unlike other QH states it possesses broken symmetry in the absence of the interlayer tunneling. It can be viewed as a pseudospin ferromagnet with the pseudospin encoding the layer degrees of freedom or as an exciton BEC with QHExs formed from electrons and holes confined to different layers.

The electric-field strength perpendicular to the layer surface gives rise to Rashba spin–orbit coupling [19, 20]. The main results of the influence of SOC on the 2D Wannier–Mott excitons in double quantum well (DQW) structures are breaking of the spin degeneracy of the electrons and holes, changes of the exciton structure, and new properties of the Bose–Einstein condensed excitons [21, 22]. In Refs. [8, 23] the energy spectrum and BEC of 2D magnetoexcitons were studied with the assumptions that the spin polarizations of electrons and holes take place and the spin–orbit coupling was neglected. In reality, as was shown in Ref. [24], the RSOC leads to breaking of the pure spin polarizations and the new spinor-type states are characterized by different numbers of Landau levels for different spin projections. These numbers for electrons differ by 1, whereas for holes they differ by 3. Spin-polarized states under the influence of the RSOC are transformed into mixed spinor components. The two lowest electron states and four lowest hole states were used to construct eight lowest 2D magnetoexciton states [24]. The electric field strength E_z depends on the density of charges in the system [25, 26]. The coupling constants for different values of the electric field were calculated in Refs. [21, 22] and it was concluded that at $E_z = 100 - 200\ \mathrm{kV\,cm^{-1}}$ the RSOC is a dominant mechanism for the energy band spin splitting. Below, we will use these results to determine the properties of the e–h system beyond the Hartree–Fock–Bogoliubov approximation, taking into account the RSOC. We consider two aspects of the problem: the influence RSOC on the chemical potential of the BEC-ed magnetoexcitons and on the energy per pair in the composition of EHL and EHD.

2 Hamiltonian of the Coulomb electron–hole interaction

The e–h Coulomb interaction we obtain below takes into account the influence of the RSOC in the frame of the conduction and valence bands. The corresponding Bloch wavefunctions for electrons including their periodic parts are [24]:

$$|\psi_c(R_1,p;x,y)\rangle = \frac{e^{ipx}}{\sqrt{L_x}} U_{c,s,p}(\boldsymbol{r}) \begin{vmatrix} a_0\varphi_0(\eta_c) \\ b_1\varphi_1(\eta_c) \end{vmatrix};$$

$$|\psi_v(R_1,q;x,y)\rangle = \frac{e^{iqx}}{\sqrt{L_x}} \frac{1}{\sqrt{2}} (U_{v,P_x,q}(\boldsymbol{r}) - iU_{v,P_y,q}(\boldsymbol{r})) \begin{vmatrix} c_3\varphi_3(\eta_v) \\ d_0\varphi_0(\eta_v) \end{vmatrix};$$

$$\eta_c = \frac{y}{l} - pl; \quad \eta_v = \frac{y}{l} - ql. \tag{1}$$

Here, $U_{c,s,p}(\boldsymbol{r})$ is the s-type conduction band periodic part and $(U_{v,P_x,q}(\boldsymbol{r}) + iU_{v,P_y,q}(\boldsymbol{r}))/\sqrt{2}$ are the p-type valence-band periodic parts. a_0, b_1, d_0, c_3 are the coefficients of the spin–orbit interaction [24]. The strong magnetic field is strong only in comparison with the exciton binding energy, but is very weak in comparison with the bandgap in GaAs-type QWs. Therefore the magnetic field in this case cannot change the periodic Bloch wavefunctions [27].

The orthogonality between the conduction and valence electron Bloch wavefunctions is attained due to their orthogonal periodic parts, whereas the orthogonality of the wavefunctions belonging to the same bands and having the same periodic parts was reached due to different numbers of Landau quantization wavefunctions $\varphi_{c,n}(y,p)$ and $\varphi_{v,m}(y,p)$. The conduction and valence electrons have the same electric charge $-|e|$ and their dimensionless variables have the same structure $\frac{y}{l} - pl$ and $\frac{y}{l} - ql$. The last variable is $\frac{y}{l} + ql$ in the case of the hole wavefunction $\varphi_{h,n}(y,q)$ due to the positive value of the hole charge $|e|$.

The electron and hole states on the given Landau level are N manifold degenerated, where

$$N = \frac{S}{2\pi l^2}; \quad l^2 = \frac{\hbar c}{eH}. \tag{2}$$

Here, S is the surface area of the layer and $2\pi l^2$ is the area of the quantum cyclotron orbit with the radius $l\sqrt{2}$. Thus, N is the total number of the possible center of gyration positions on the surface of the layer.

The Hamiltonian of the Coulomb e–h interaction can be calculated in the same way as was demonstrated in Ref. [8]. By contrast, we had dealt with spinor-type wavefunctions for electrons and holes with a column representation (1). The creation and annihilation operators for conduction and valence electrons are denoted as $a^\dagger_{c,R_i,p}, a_{c,R_i,p}, a^\dagger_{v,R_j,q}, a_{v,R_j,q}$, where p and q are the momentum. The Hamiltonian of their Coulomb

interaction has the form:

$$\begin{aligned}\hat{H}=&-\mu_e\sum_p a^\dagger_{c,R_i,p}a_{c,R_i,p}-\mu_h\sum_q a^\dagger_{v,R_j,q}a_{v,R_j,q}\\&+\frac{1}{2}\sum_{p,q,s}F_{c-c}(c,R_i,p;c,R_i,q;c,R_i,p-s;c,R_i,q+s)\\&\qquad\times a^\dagger_{c,R_i,p}a^\dagger_{c,R_i,q}a_{v,R_i,q+s}a_{c,R_i,p-s}\\&+\frac{1}{2}\sum_{p,q,s}F_{v-v}(v,R_j,p;v,R_j,q;v,R_j,p-s;v,R_j,q+s)\\&\qquad\times a^\dagger_{v,R_j,p}a^\dagger_{v,R_j,q}a_{v,R_j,q+s}a_{v,R_j,p-s}\\&-\sum_{p,q,s}F_{c-v}(c,R_i,p;v,R_j,q;c,R_i,p-s;v,R_j,q+s)\\&\qquad\times a^\dagger_{c,R_i,p}a^\dagger_{v,R_j,q}a_{v,R_j,q+s}a_{c,R_i,p-s}.\end{aligned}\tag{3}$$

Here, the first two terms represent noninteracting electrons and holes, the Coulomb matrix elements defined as follows:

$$\begin{aligned}&F_{c-v}(c,R_i,p;v,R_j,q;c,R_i,p-s;v,R_j,q+s)\\&=\int d\rho_1\int d\rho_2\;\psi^{c*}_{R_i,p}(\rho_1)\psi^{v*}_{R_j,q}(\rho_2)V_{12}\\&\qquad\times\psi^{c}_{R_i,p-s}(\rho_1)\psi^{v}_{R_j,q+s}(\rho_2),\end{aligned}\tag{4}$$

where

$$V_{12}=\frac{e^2}{\varepsilon_0|\rho_1-\rho_2|}.\tag{5}$$

Here, the function $\psi^c_{R_i,p}$ are the envelope parts (1) of the Bloch functions. Their periodic parts being integrated on the elementary lattice cell, can be excluded from the final expressions of the desirable matrix elements, because the dipole moments of the band-to-band transitions are not needed now. The variables ρ_i are 2D vectors and ε_0 is the dielectric constant of the medium. The series expansion of the Coulomb interaction

$$V_{12}=\sum_\kappa V_\kappa\exp[i\kappa(\rho_1-\rho_2)]\tag{6}$$

will be used, where $\kappa=(\kappa_x,\kappa_y)$ is the 2D wave vector and

$$V_\kappa=V_{\kappa_x,\kappa_y}=\frac{2\pi e^2}{\varepsilon_0 S|\kappa|}=\frac{2\pi e^2}{\varepsilon_0 S\sqrt{\kappa_x^2+\kappa_y^2}}.\tag{7}$$

The matrix elements (3) have the following form:

$$\begin{aligned}&\sum_{p,q,s}F_{c-c}(c,R_i,p;c,R_i,q;c,R_i,p-s;c,R_i,q+s)\\&=\sum_{\kappa_y}V_{s,\kappa_y}\exp\left[-\frac{(s^2+\kappa_y^2)l^2}{2}-i\kappa_y(q+s-p)l^2\right]\\&\times\left(|a_0|^2+\left(1-\frac{(s^2+\kappa_y^2)l^2}{2}\right)|b_1|^2\right)^2;\end{aligned}$$

$$\begin{aligned}&\sum_{p,q,s}F_{v-v}(v,R_j,p;v,R_j,q;v,R_j,p-s;v,R_j,q+s)\\&=\sum_{\kappa_y}V_{s,\kappa_y}\exp\left[-\frac{(s^2+\kappa_y^2)l^2}{2}-i\kappa_y(q+s-p)l^2\right]\times\left(|d_0|^2\right.\\&\left.+\left(1-\frac{(s^2+\kappa_y^2)l^2(-12+(s^2+\kappa_y^2)l^2)(-6+(s^2+\kappa_y^2)l^2)}{48}\right)|c_3|^2\right)^2;\end{aligned}\tag{8}$$

$$\begin{aligned}&\sum_{p,q,s}F_{c-v}(c,R_i,p;v,R_j,q;c,R_i,p-s;v,R_j,q+s)\\&=\sum_{\kappa_y}V_{s,\kappa_y}\exp\left[-\frac{(s^2+\kappa_y^2)l^2}{2}+i\kappa_y(q+p)l^2\right]\\&\times\left(|c_3|^2|a_0|^2\left(1-\frac{(s^2+\kappa_y^2)l^2(-12+(s^2+\kappa_y^2)l^2)(-6+(s^2+\kappa_y^2)l^2)}{48}\right)\right.\\&+|d_0|^2|b_1|^2\frac{(2-(s^2+\kappa_y^2)l^2)}{2}+|a_0|^2|d_0|^2+|c_3|^2|b_1|^2\left(1-2(s^2+\kappa_y^2)l^2\right.\\&\left.\left.+\frac{108(s^2+\kappa_y^2)^2l^4-20(s^2+\kappa_y^2)^3l^6+(s^2+\kappa_y^2)^4l^8}{96}\right)\right).\end{aligned}$$

Further, for simplicity, we will replace $a^\dagger_{c,R_i,p}a_{c,R_i,p}$ to $a^\dagger_p a_p$ and $a^\dagger_{v,R_j,q}a_{v,R_j,q}$ to $b^\dagger_p b_p$.

3 Bose–Einstein condensation of magnetoexcitons and metallic-type electron–hole liquid Consider the BEC of magnetoexcitons in a single particle state with wave vector $\boldsymbol{k}$ in the HFBA and Rashba spin–orbit interaction. The energy of electrons and holes as well as their chemical potentials are measured relative to their LLLs. The exciton formation reaction $e+h\leftrightarrow ex$ implies the relation between the chemical potentials

$$\mu_e+\mu_h=\mu_{ex}=\mu.\tag{9}$$

In the case of BEC of magnetoexcitons in Ref. [6] as a ground-state wave function was chosen the BCS-type wave function $|\psi_g(k)\rangle$ and as the excited-wave functions the wave functions of the coherent excited states introduced in Ref. [6] for e–h systems in a similar way as was done by Anderson [28] in the case of superconductors. The ground-state wave function was introduced following the Keldysh–Kozlov method [29] by the action of the displacement unitary transformation $\hat{D}(\sqrt{N_{ex}})$ on the vacuum state of the initially

introduced e–h operators

$$\left|\psi_g(k)\right\rangle = \hat{D}(\sqrt{N_{ex}})|0\rangle; \quad a_p|0\rangle = b_p|0\rangle = 0. \tag{10}$$

The coherent excited states were generated in Ref. [6] by the expression

$$\left|\psi^e\left(q \pm \frac{Q_x}{2}\right)\right\rangle = a_{q+\frac{Q_x}{2}} a_{q-\frac{Q_x}{2}} \left|\psi_g(k)\right\rangle. \tag{11}$$

The unitary transformation $\hat{D}(\sqrt{N_{ex}})$ breaks the gauge symmetry of the starting Hamiltonian (3) and transforms it into a new Hamiltonian $\hat{D}H\hat{D}^{\dagger}$, giving the ground-state wave function $\left|\psi_g(k)\right\rangle$ and the macroscopic displacement $\sqrt{N_{ex}}$ of the exciton creation operator,

$$d^{\dagger}(k) = \frac{1}{\sqrt{N}} \sum_t e^{-iQ_y t l^2} a^{\dagger}_{\frac{k_x}{2}+t} b^{\dagger}_{\frac{k_x}{2}-t}. \tag{12}$$

In contrast to the quantum optics and Glauber coherent states [30] the exciton creation and annihilation operators are not pure Bose operators but only quasiboson operators [31].

The unitary transformation

$$\hat{D}\left(\sqrt{N_{ex}}\right) = \exp\left[\sqrt{N_{ex}}(d^{\dagger}(k) - d(k))\right] \tag{13}$$

leads to a Bogoliubov u–v transformation

$$\begin{aligned} \alpha_p &= Da_pD^{\dagger} = ua_p - v\left(p - \frac{k_x}{2}\right) b^{\dagger}_{k_x - p} \\ \beta_p &= Db_pD^{\dagger} = ub_p + v\left(\frac{k_x}{2} - p\right) a^{\dagger}_{k_x - p}, \end{aligned} \tag{14}$$

as well as to inverse transform action

$$\begin{aligned} a_p &= u\alpha_p + v\left(p - \frac{k_x}{2}\right) \beta^{\dagger}_{k_x - p} \\ b_p &= u\beta_p - v\left(\frac{k_x}{2} - p\right) \alpha^{\dagger}_{k_x - p}, \end{aligned} \tag{15}$$

where

$$u = \cos g, \quad v = \sin g, \quad u^2 + v^2 = 1,$$

$$v(t) = ve^{-iK_y t l^2}; \quad g = \sqrt{2\pi l^2 n_{ex}}; \quad n_{ex} = \frac{N_{ex}}{S}, \tag{16}$$

with the confinement of the LLLs. It was shown that [6]

$$N_{ex} = v^2 N; \quad n_{ex} = \frac{v^2}{2\pi l^2}; \quad g = v, \tag{17}$$

where v^2 is the filling factor of the LLLs.

This leads to the relations $u = \cos v$ and $v = \sin v$, which can be satisfied only in the limit $v < 1$. The theory developed in Ref. [6] and its application below can be made in the restriction $v < 1$.

The Hamiltonian (3) after the unitary transformation (15) will contain operators $\alpha^{\dagger}_p, \alpha_p, \beta^{\dagger}_p, \beta_p$ in arbitrary order:

$$H = E_g + H_2 + H'. \tag{18}$$

The first term E_g does not contain operators α_p and β_p and plays the role of the new ground-state energy. The second term H_2 is quadratic in the operators α_p and β_p and appears as a result of transpositions of the new operators and their normal ordering. In this transposition, the commutation relations of the Fermi operators α_p and β_p transform terms with four operators into quadratic terms. The term H' contains the remaining normal-ordered terms with four operators, which is treated as a perturbation. The term E_g can be represented as

$$\begin{aligned} E_g = &-\mu N_{ex} - (1 - v^2) N_{ex} I_{ex}(\boldsymbol{k}) |a_0|^2 |d_0|^2 \\ &- \frac{1}{2} v^2 N_{ex} I_l |a_0|^4 - \frac{1}{2} v^2 N_{ex} I_l |d_0|^4 \\ &- \frac{3}{8} v^2 N_{ex} I_l |b_1|^4 - \frac{147}{512} v^2 N_{ex} I_l |c_3|^4, \end{aligned} \tag{19}$$

where $I_{ex}(\boldsymbol{k})$ is the exciton ionization potential:

$$I_{ex}(\boldsymbol{k}) = I_l e^{-k^2 l^2/4} I_0\left(\frac{k^2 l^2}{4}\right).$$

Here, I_l is the ionization potential of magnetoexciton within the LLLs approximation and equals $(e^2/\varepsilon l)\sqrt{\pi/2}$, where l is magnetic length and ε is the background dielectric constant. $I_0(z)$ is the modified Bessel function.

The term H_2 contains diagonal quadratic terms as well as the terms describing the creation and annihilation of the new e–h pairs from the new vacuum state $\left|\psi_g(\boldsymbol{k})\right\rangle$. It has the form

$$\begin{aligned} H_2 = &\sum_p E(\boldsymbol{k}, v^2, \mu)(\alpha^{+}_p \alpha_p + \beta^{+}_p \beta_p) \\ &- \sum_p [uv\left(\frac{k_x}{2} - p\right) \beta_{k_x - p} \alpha_p \\ &+ uv\left(p - \frac{k_x}{2}\right) \alpha^{+}_p \beta^{+}_{k_x - p}] \psi(\boldsymbol{k}, v^2, \mu), \end{aligned} \tag{20}$$

where

$$\begin{aligned} E(\boldsymbol{k}, v^2, \mu) = &-u^2 v^2 I_l |a_0|^4 - \frac{3}{4} u^2 v^2 I_l |b_1|^4 \\ &+ v^4 I_l |d_0|^4 + \frac{147}{256} I_l v^4 |c_3|^4 \\ &+ 2u^2 v^2 I_{ex}(\boldsymbol{k}) |a_0|^2 |d_0|^2 - \frac{\mu}{2}(u^2 - v^2) \end{aligned} \tag{21}$$

and

$$\begin{aligned} \psi(\boldsymbol{k}, v^2, \mu) = &-I_{ex}(\boldsymbol{k})(v^2 - u^2) |a_0|^2 |d_0|^2 \\ &+ \frac{3}{4} I_l v^2 \left(\frac{49}{64} |c_3|^4 + |b_1|^4\right) + I_l v^2 \left(|a_0|^4 + |d_0|^4\right) + \mu. \end{aligned} \tag{22}$$

It is seen from the Hamiltonian H_2 that the new quasiparticles described by the operators α_p, β_p can appear spontaneously from the new vacuum state as a pair with total momentum k_x, which coincides with the translational wave vector of the Bose–Einstein condensate of magnetoexcitons.

Such terms in the Hamiltonian and the corresponding diagrams are called dangerous ones and make the new vacuum state unstable. The method of compensation of dangerous diagrams proposed by Bogolyubov in 1957 allowed to give a mathematical justification of the phenomenological premises of the previous theories of superconductivity and also led to the important concept of collective electron excitations, described by the canonical transformation of electron operators. To obtain specific results of the theory of superconductivity was sufficient to compensate for the dangerous diagrams corresponding to creation from vacuum of two electron excitations with opposite momenta and spins, *i.e.*, dangerous electron bivertex [32, 33]. To avoid this instability, the condition of compensation of the dangerous diagrams is used. In the HFBA, when only the dangerous diagrams in H_2 are taken into account, the condition of their compensation is

$$\psi(\boldsymbol{k}, v^2, \mu) = 0. \tag{23}$$

This condition determines the unknown parameter of the theory, namely the chemical potential μ of the system. In the HFBA it is

$$\mu = I_{ex}(\boldsymbol{k})(v^2 - u^2)|a_0|^2|d_0|^2 - \frac{3}{4}I_l v^2\left(\frac{49}{64}|c_3|^4 + |b_1|^4\right) - I_l v^2\left(|a_0|^4 + |d_0|^4\right). \tag{24}$$

With the help of μ we can determine self-consistently the ground-state energy E_g and the energy of the single-particle elementary excitations, which in the given approximation are:

$$\begin{aligned} E_g &= v^2 N_{ex}\left(\frac{1}{2}I_l\left(|a_0|^4 + |d_0|^4\right) - I_{ex}(\boldsymbol{k})|a_0|^2|d_0|^2\right) \\ &\quad + \frac{3}{8}v^2 N_{ex} I_l\left(\frac{49}{64}|c_3|^4 + |b_1|^4\right), \\ E(\boldsymbol{k}, v^2, \mu) &= \frac{1}{2}I_{ex}(\boldsymbol{k}) - \frac{1}{2}I_l v^2|a_0|^4 \\ &\quad + \frac{1}{2}I_l v^2|d_0|^4 + \frac{3}{8}\left(\frac{49}{64}|c_3|^4 - |b_1|^4\right)I_l v^2. \end{aligned} \tag{25}$$

Now we consider the EHL formation in Hartree–Fock approximation (HFA). We start with the Hamiltonian (3), but without chemical potentials μ_e and μ_h, and calculate the ground-state energy E_{EHL} of EHL at $T = 0$ when the average values of electrons and holes numbers are equal to

$$\left\langle a_p^\dagger a_p \right\rangle = \left\langle b_p^\dagger b_p \right\rangle = v^2. \tag{26}$$

Here, v^2 is the filling factor. Applying the Wick theorem, we obtained the ground-state energy

$$\begin{aligned} E_{\mathrm{EHL}} &= -\frac{N_{\mathrm{e\text{-}h}}}{2}v^2 I_l\left[\frac{4|a_0|^4 + 4|a_0|^2|b_1|^2 + 3|b_1|^4}{4} + \frac{256|d_0|^4 + 160|d_0|^2|c_3|^2 + 147|c_3|^4}{256}\right]; \\ N_{\mathrm{e\text{-}h}} &= v^2 N, \end{aligned} \tag{27}$$

and the energy per one e–h pair E_{EHL} in units of I_l

$$\frac{\varepsilon_{\mathrm{EHL}}}{I_l} = \frac{E_{\mathrm{EHL}}}{N_{\mathrm{e\text{-}h}}} = -\frac{v^2}{2}\left[\frac{4|a_0|^4 + 4|a_0|^2|b_1|^2 + 3|b_1|^4}{4} + \frac{256|d_0|^4 + 160|d_0|^2|c_3|^2 + 147|c_3|^4}{256}\right]. \tag{28}$$

The minimal value is achieved at filling factor $v^2 = 1$, and it determines the energy per pair inside EHD equal to

$$\varepsilon_{\mathrm{EHD}} = -\frac{1}{2}\left[\frac{4|a_0|^4 + 4|a_0|^2|b_1|^2 + 3|b_1|^4}{4} + \frac{256|d_0|^4 + 160|d_0|^2|c_3|^2 + 147|c_3|^4}{256}\right]. \tag{29}$$

This result agrees well with the result of Ref. [6]. Indeed, if we assume that there is no spin-orbit interaction then from Ref. [24] $|a_0|^2 = |d_0|^2 = 1$ and $|c_3|^2 = |b_1|^2 = 0$, and we will get exactly the same expression for the chemical potential, the ground-state energy E_g and the energy of the single-particle elementary excitations as in Ref. [6].

One can remember that in the GaAs-type crystal the electron cyclotron energy $\hbar\omega_{ce}$ becomes comparable to and larger than the 2D Wannier–Mott exciton binding energy and at the same time the magnetic length l becomes smaller than the exciton Bohr radius just at the values $H \geq 7$ T.

Magnetoexcitons exist only in the range of a highly perpendicular magnetic field. Therefore, we will demonstrate our results with high magnetic and electric fields: $E_z = 10\ \mathrm{kV\,cm^{-1}}$ and $H = 10$ T.

Figure 1 presents the chemical potential versus filling factor v^2 for different values of the wave vector $\boldsymbol{k}$ and the energy per e–h pair of EHD. It can be seen that the chemical potential is a monotonic function with negative compressibility, which leads to instability of the Bose–Einstein condensate of magnetoexcitons. A similar result was

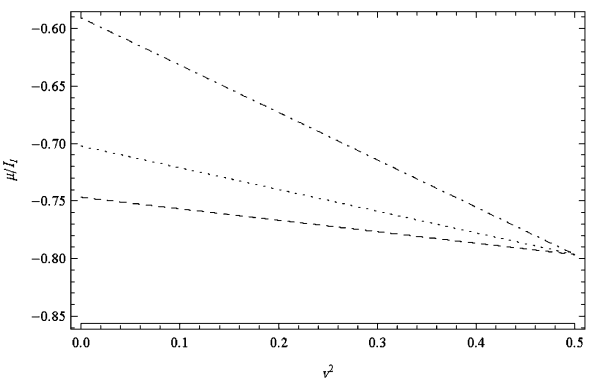

Figure 1 Chemical potential in units of exciton binding energy I_l versus filling factor v^2. Solid line: energy per e–h pair in EHD phase; dashed line: chemical potential of condensed excitons with $kl = 0$; dotted line: the same, but for $kl = 0.5$; dash-dotted line: the same, but for $kl = 1$.

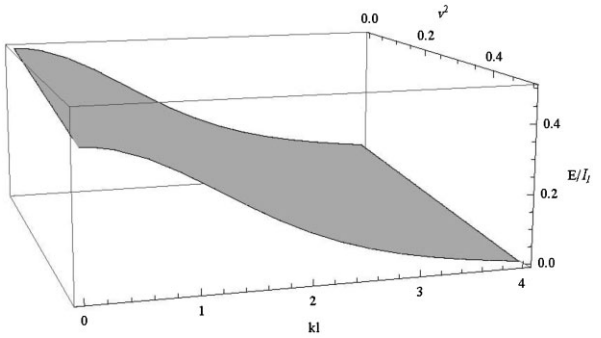

Figure 2 The energy of the single-particle elementary excitations versus filling factor v^2 and wave vector $\boldsymbol{k}$.

obtained in Ref. [6], when the BEC of magnetoexcitons was considered without taking into account RSOC and excited Landau levels. The energy of an *e–h* pair of EHD is situated on the energy scale lower than the value of the chemical potential of condensed excitons with small wave vectors $\boldsymbol{k}$ and EHL state is more preferable than BEC of magnetoexcitons.

The energy of the single-particle elementary excitations and the ground-state energy depends on the two parameters: one of them is the filling factor v^2, and the other is the wave vector $\boldsymbol{k}$. Their behavior can be seen in Figs. 2 and 3. On increasing filling of the wavevector energy of the single-particle elementary excitations decreases asymptotically. The ground-state energy has a reverse picture with increasing values of the wavevector ground-state energy increasing with saturation. Also, it can be noted that if the ground-state energy increases with the increases filling factor, but the energy of the single-particle elementary excitations decreases.

4 Conclusions The *e–h* system in an ideal symmetric 2D layer in a strong perpendicular magnetic field are studied taking into account the Rashba spin–orbit interaction. This influence on the chemical potential of the BEC-ed magnetoexcitons and on the energy per pair in the components of EHL and EHD were studied in the HFA. We have established that chemical potential is a monotonic function versus the value of the filling factor with negative compressibility, which leads to instability of the Bose–Einstein condensate of magnetoexcitons.

The energy per *e–h* pair inside the EHD was found to be situated on the energy scale lower than the value of the chemical potential of the Bose–Einstein condensed magnetoexcitons with wave vector $\boldsymbol{k} = 0$ calculated in the HFA. Therefore, the EHL state is more preferable than the BEC magnetoexcitons. Our results well agree with Ref. [6]. Indeed, if we make the limiting transition and put RSOC equal to zero, which means that the coefficients of RSOC will be $|a_0|^2 = |d_0|^2 = 1$ and $|b_1|^2 = |c_3|^2 = 0$, and consequently Eqs. (24), (25), (28), (29) exactly coincide with the results of Ref. [6].

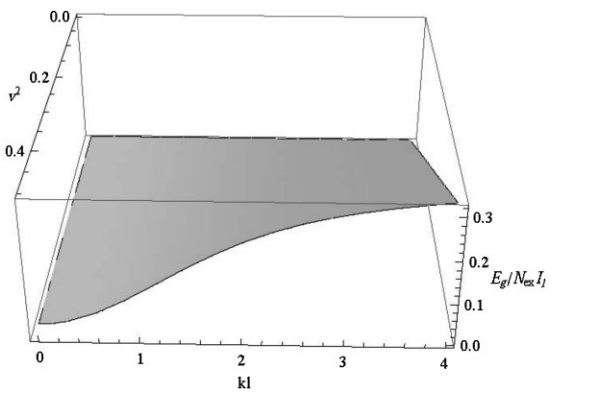

Figure 3 The ground-state energy versus filling factor v^2 and wave vector $\boldsymbol{k}$.

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