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Mixed exciton–plasmon collective elementary excitations of the Bose–Einstein condensed two-dimensional magnetoexcitons with motional dipole moments

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The collective elementary excitations of the two-dimensional (2D) magnetoexcitons in the state of their Bose–Einstein condensation (BEC) with nonzero wave vector $\boldsymbol{k}$ and inplane parallel oriented motional dipole moments are investigated in the Hartree–Fock–Bogoliubov approximation (HFBA). The breaking of the gauge symmetry is achieved using the Bogoliubov theory of quasiaverages and the Keldysh–Kozlov–Kopaev (KKK) method. The starting Hamiltonian and the Green's functions are determined using the integral two-particle operators instead of the single-particle Fermi operators. The infinite chains of equations of motion for the multioperator four- and six-particle Green-s functions are truncated following the Zubarev method and introducing a small parameter of the perturbation theory related with the lowest Landau levels (LLLs) filling factor and with the phase-space filling factor. The energy spectrum of the collective elementary excitations consists of the mixed exciton–plasmon energy braches, mixed exciton–plasmon quasienergy branches as well as the optical and acoustical plasmon energy branches. The exciton branches of the spectrum have gaps related with the negative values of the chemical potential and attractive interaction between the 2D megnetoexcitons with inplane, parallel oriented motional dipole moments. The slopes of the mixed exciton–plasmon branches are determined by the group velocities of the moving condensed excitons in the laboratory reference frame. The acoustical and optical plasmon energy branches are gapless. Their dependence on the small wave vectors accounted from the condensate wave vector $\boldsymbol{k}$ is linear and quadratic, respectively, with saturation in the range of high values of the wave vectors.

1 Introduction The properties of the symmetric two-dimensional (2D) electron–hole (e–h) system, with equal concentrations of both components in a strong perpendicular magnetic field, with coinciding matrix elements of Coulomb electron–electron (e–e), hole–hole (h–h), and e–h interactions have attracted a great deal of attention in the last two decades [1–8]. A hidden symmetry and the multiplicative states were discussed in numerous papers [5, 9, 10]. The collective states such as the Bose–Einstein condensation (BEC) of 2D magnetoexcitons and the formation of the metallic-type electron–hole liquid (EHL) were investigated in Refs. [1–8]. Studying the phenomenon of the BEC has become a milestone in condensed-matter physics [11]. The remarkable properties of superfluids and superconductors are intimately related to the existence of a bosonic condensate of composite particles consisting of an even number of fermions. In highly excited semiconductors the role of such composite bosons is taken on by excitons, which are the bound states of electrons and holes. Furthermore, the excitonic system has been viewed as a keystone system for exploration of the BEC phenomena, since it allows the particle density and interaction to be controlled *in situ*. Promising candidates for experimental realization of such system are semiconductor quantum wells (QWs) [12], which

have a number of advantages compared to the bulk systems. The coherent pairing of electrons and holes occupying only the lowest Landau levels (LLLs) has been studied using the Keldysh–Kozlov–Kopaev (KKK) method and the generalized random-phase approximation (RPA) in Refs. [6, 13]. The BEC of magnetoexcitons takes place in a single exciton state with the wave vector $\boldsymbol{k}$, suggesting that the high density of electrons in the conduction band and of holes in the valence band were created in a single QW structure with the size quantization much greater than the Landau quantization. When $\boldsymbol{k} \neq 0$ a new metastable dielectric liquid phase formed by Bose–Einstein condensed magnetoexcitons has been revealed [6, 7]. The importance of the excited Landau levels (ELLs) and their influence on the ground states of the systems was first noted in Refs. [2–5]. The influence of the ELLs of electrons and holes was studied in detail in Refs. [7, 8]. The indirect attraction between electrons (e–e), between holes (h–h) and between electrons and holes (e–h) due to the virtual simultaneous quantum transitions of the interacting charges from LLLs to ELLs is a result of their Coulomb scattering. The first step of the scattering and the return to the initial states were described in the second order of the perturbation theory.

Plasmon oscillations in the one-component system of the monolayer in a strong perpendicular magnetic field were studied by Girvin et al. [14], who proposed the magnetoroton theory of collective excitations for the conditions of the fractional quantum Hall effect (FQHE). The FQHE occurs in the low-disorder, high-mobility samples with partially filled Landau levels with the filling factor in the form $\nu = 1/m$, where m is an integer. Considerable progress has recently been achieved toward understanding the nature of the many-body ground-state well described by the Laughlin variational wave function [15]. Theory of the collective excitation spectrum proposed in Ref. [14] is closely analogous to Feynman's theory of superfluid helium [16]. The main Feynman arguments lead to the conclusions that on general grounds the low-lying excitations of any system will include density waves. As regards the 2D system the perpendicular magnetic field quenches the single-particle continuum of the kinetic energy leaving a series of discrete highly degenerate Landau levels, which are spaced in energy at intervals $\hbar\omega_c$. In the case of the filled Landau level, $\nu = 1$, the lowest excitation is necessarily the cyclotron mode in which particles are excited into the next Landau level, because of the Pauli exclusion principle. In the case of FQHE the LLL is fractionally filled. The Pauli principle no longer excludes low-energy intra-Landau-level excitations. For the FQHE case the low-lying excitations have primary importance, rather than the high-energy inter-Landau-level cyclotron modes [14]. The spectrum has a relatively large excitation gap at zero wave vector $\boldsymbol{kl} = 0$ and in addition it exhibits a deep magnetoroton minimum at $\boldsymbol{kl} \sim 1$, quite analogous to the roton minimum in helium. The magnetoroton minimum becomes deeper with decreasing filling factor ν in the row $1/3,\ 1/5,\ 1/7$ and this is a precursor of the gap collapse associated with the Wigner crystallization that occurs at $\nu = 1/7$. For the largest wave vectors the low-lying mode crosses over from being a density wave to becoming a quasiparticle excitation [14]. The Wigner crystal transition occurs slightly before the roton mode becomes completely soft. The magnitude of the primitive reciprocal lattice vector for the crystal lies close to the position of the magnetoroton minimum. The authors of Ref. [14] suggested also the possibility of pairing of two rotons of opposite momenta leading to the coupling of two-roton states with a small total momentum, as is known to occur in helium. Contrary to the case of fractional filling factor, the excitations of a filled Landau level in the 2DEG were studied by Kallin and Halperin [17].

Fertig [18] investigated the excitation spectrum of two-layer and three-layer electron systems. In particular the two-layer system in a strong perpendicular magnetic field with filling factor $\nu = 1/2$ of the LLL in the conduction band of each layer was considered. The interlayer separation z and spontaneous coherence of a two-component 2D electron gas were introduced.

Both half-filled layers a and b are accompanied by a substrate with the positive charge, which ensures the electrical neutrality of the system. The half-filled layer a can be considered as fully filled by electrons in the LLL of the conduction band and a half-filled by holes in the same LLL of the conduction band.

The electrons of the fully filled conduction band are compensated by the charge of the substrate and we can only consider the electrons on the layer b and the holes on the layer a. Then, the wave function [18] of the coherent two-layer electron system can be rewritten in the form that coincides with the BCS-type wave function of the superconductor. It represents the coherent pairing of the conduction electrons on the LLL of the layer b with the holes in the LLL of the conduction band of the layer a and describes the BEC of such unusual excitons named FQHE excitons, because they appear under conditions suitable for observation of the FQHE. Here, only the BEC on the single exciton state with wave vector $\boldsymbol{k} = 0$ is considered.

Fertig determined the energy spectrum of the elementary excitations in the framework of this ground state. In the case when $z = 0$ the lowest-lying excitations of the system are the higher-energy excitons.

Because of the neutral nature of the $\boldsymbol{k} = 0$ excitons the dispersion relation of these excitations is to a good approximation given by $\hbar\omega(\boldsymbol{k}) = E_{\mathrm{ex}}(\boldsymbol{k}) - E_{\mathrm{ex}}(0)$, where $E_{\mathrm{ex}}(\boldsymbol{k})$ is the energy of the exciton with wave vector $\boldsymbol{k}$. This result was first obtained by Paquet et al. [5] using a RPA. For $z = 0$ the dispersion relation $\omega(\boldsymbol{k})$ vanishes as $\boldsymbol{k}^2$ for $k \to 0$, as one expects for the Goldstone modes.

$\omega(\boldsymbol{k})$ behaves as an acoustical mode $\omega(\boldsymbol{k}) \sim \boldsymbol{k}$ in the range of small $\boldsymbol{k}$ for $z > 0$, whereas $\omega(\boldsymbol{k})$ tends to the ionization potential $\Delta(z)$ in the limit $k \to \infty$.

In the region of intermediate values of $\boldsymbol{k}$, when $\boldsymbol{kl} \sim 1$, the dispersion relation develops dips as z increases. At a certain critical value of $z = z_{\mathrm{cr}}$ the modes in the vicinity of the minima become equal to zero and are named soft modes.

Their appearance indicates that the two-layer system undergoes a phase transition to the Wigner crystal state.

Similar results concerning the linear and quadratic dependences of the dispersion relations in the range of small wave vectors $\boldsymbol{q}$ were obtained by Kuramoto and Horie [19], who studied the coherent pairing of electrons and holes spatially separated by the insulator barrier in the structure of the type coupled double quantum wells (CDQW).

The appearance of the soft modes in the spectrum of the collective elementary excitations may signal not only about the possible phase transition of the two-layer system to the Wigner crystal state or to the charge-density-wave (CDW) of a 2D electron system, but also to another variant of the excitonic charge-density-wave (ECDW) state. This new state was revealed theoretically by Chen and Quinn [20, 21], who studied the ground state and the collective elementary excitations of the system consisting of spatially separated electron and hole layers in a strong magnetic field. When the interlayer Coulomb attraction is strong, the electrons and holes are paired to form excitons. Excitonically condensed state of e–h pairs is the preferable ground state. If the layer separation is larger than the critical value, a novel excitonic-density-wave state is found to have a lower energy than either a homogeneous exciton fluid or a double CDW state in a 2D electron system.

Of interest is the case of $\boldsymbol{k} \neq 0$ with filling factor $\nu = \nu^2 < 1$ because in this range of parameters a metastable dielectric liquid phase exists. It is formed by the Bose–Einstein condensed magnteoexcitons with $\boldsymbol{kl} \sim 3 - 4$ and with nonzero motional dipole moments $\rho = [\boldsymbol{k} \times \boldsymbol{z}]l^2$. This state was revealed in Ref. [6], where the system of electrons and holes on their LLLs was studied, without addressing the ELLs, but taking into account coherent excited states, when one e–h pair exits from the condensate leaving all another pairs in their coherent pairing state. The correlation energy was calculated beyond the Hartree–Fock–Bogoliubov approximation (HFBA) in the framework of the KKK method using the Nozieres Comte approach [6, 22].

The Bose–Einstein condensed magnetoexcitons moving as a whole with wave vector $\boldsymbol{k}$ and with parallel motional dipole moments $\boldsymbol{\rho}$ have a significant polarizability that gives rise to the attractive interaction between them and to the lower values of the chemical potential and the mean energy per one e–h pair. This lowering is not monotonous and at some value of the filling factor ν_{m}^2 the relative minimum on the corresponding curves appears with positive compressibility in their vicinity. The relative minimum on the chemical potential curve depends essentially on the damping of the magnetoexciton level. The problem in question was investigated in Ref. [7] and is represented in Fig. 1.

If the average filling factor ν^2 is less than ν_{m}^2 the dielectric liquid phase exists in the form of droplets with optimal concentration $n_{\mathrm{ex}} = \nu_{\mathrm{m}}^2/2\pi l^2$ inside them, corresponding to the filling factor ν_{m}^2.

Following the ideas proposed by Bogoliubov in his theory of quasiaverages [23] in the present paper we will break the gauge symmetry of the starting Hamiltonian. Such

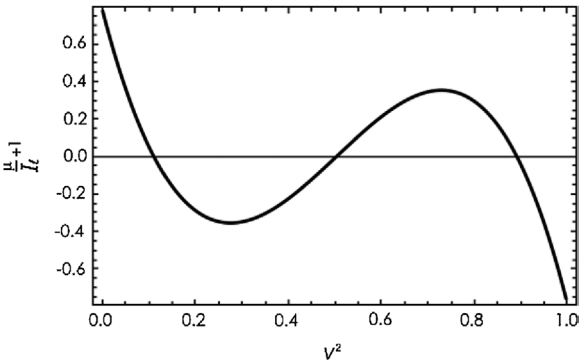

Figure 1 The relative minimum on the chemical potential depending on the filling factor.

an approach is equivalent and supplementary to the KKK method [13] and has the same starting unitary transformation introducing as a ground state of the system the coherent macroscopic state $|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle$. As usual, the next step is the successive Bogoliubov u–v transformation from the initial electron and hole Fermi operators a_p, b_p to the new quasiparticle Fermi operators α_p, β_p in all parts of the calculations. We will follow this only when the chemical potential in the HFBA is determined and the average values of some operators is calculated using the ground-state wave function $|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle$. The collective elementary excitations of the system studied beginning with the equations of motion for the two-particle integral operators such as exciton creation and annihilation operators $d^{\dagger}(\boldsymbol{p})$ and $d(\boldsymbol{p})$ accompanied by two electron–hole inphase and out-of-phase density operators $D(\boldsymbol{p})$ and $\rho(\boldsymbol{p})$, respectively. These four two-particle integral operators form a closed set because the commutators between them as well as their commutations with the Hamiltonians can be expressed as the functions of the same four basic two-particle integral operators $d^{\dagger}(\boldsymbol{p})$, $d(\boldsymbol{p})$, $D(\boldsymbol{p})$, and $\rho(\boldsymbol{p})$.

This is so because the starting Hamiltonian as well as the Hamiltonian with a broken gauge symmetry can be expressed in terms of the same basic integral two-particle operators even in the case when the influence of the ELLs is taken into account together with the Coulomb interaction of the electrons and holes lying on the LLLs. This allows us to consider the equations of motion for the products of the four basic operators $d^{\dagger}(\boldsymbol{p})$, $d(\boldsymbol{p})$, $D(\boldsymbol{p})$, and $\rho(\boldsymbol{p})$ in any combinations and to obtain the infinite chains of equations of motion, which appear due to the nonlinearities and interactions in the system. Many-particle complexes, such as bi-, tri-, and quadriexciton states [24] can be also described in this way.

In the present paper we restrict ourselves to the exciton and plasmon types elementary excitations. It is based on the combination of two alternative but closely related methods and is organized as follows. Section 2 is dedicated to the breaking of the gauge symmetry of the initial Hamiltonian and to the equations of motion for many-particle integral operators and the corresponding Green's functions. The

energy spectrum of the exciton and plasmon types collective elementary excitations are described in collinear geometry of observation in Section 3. The obtained results are summarized in the concluding section.

2 The breaking of the gauge symmetry of the initial Hamiltonian: Two equivalent representations First, we will introduce the operators describing the magnetoexcitons, the plasmons, and their commutation relations. The creation and annihilation operators of magnetoexcitons are two-particle operators reflecting the e–h structure of the excitons. They are denoted below as $d^{\dagger}(\boldsymbol{p})$ and $d(\boldsymbol{p})$, where $\boldsymbol{p}(p_x, p_y)$ is the 2D wave vector. There are also the density fluctuation operators for electrons $\hat{\rho}_{\mathrm{e}}(\boldsymbol{Q})$ and for holes $\hat{\rho}_{\mathrm{h}}(\boldsymbol{Q})$ as well as their linear combinations $\hat{\rho}(\boldsymbol{Q})$ and $\hat{D}(\boldsymbol{Q})$, which are expressed through the Fermi creation and annihilation operators $a_p^{\dagger}, a_p$ for electrons and $b_p^{\dagger}, b_p$ for holes as follows

$$\begin{aligned}
\hat{\rho}_{\mathrm{e}}(\boldsymbol{Q}) &= \sum_t e^{iQ_y t l^2} a^{\dagger}_{t-\frac{Q_x}{2}} a_{t+\frac{Q_x}{2}};\\
\hat{\rho}_{\mathrm{h}}(\boldsymbol{Q}) &= \sum_t e^{iQ_y t l^2} b^{\dagger}_{t+\frac{Q_x}{2}} b_{t-\frac{Q_x}{2}};\\
\hat{\rho}(\boldsymbol{Q}) &= \hat{\rho}_{\mathrm{e}}(\boldsymbol{Q}) - \hat{\rho}_{\mathrm{h}}(-\boldsymbol{Q});\\
\hat{D}(\boldsymbol{Q}) &= \hat{\rho}_{\mathrm{e}}(\boldsymbol{Q}) + \hat{\rho}_{\mathrm{h}}(-\boldsymbol{Q});\\
d^{\dagger}(\boldsymbol{P}) &= \frac{1}{\sqrt{N}} \sum_t e^{-iP_y t l^2} a^{\dagger}_{t+\frac{P_x}{2}} b^{\dagger}_{-t+\frac{P_x}{2}};\\
d(\boldsymbol{P}) &= \frac{1}{\sqrt{N}} \sum_t e^{iP_y t l^2} b_{-t+\frac{P_x}{2}} a_{t+\frac{P_x}{2}};
\end{aligned} \tag{1}$$

$$\begin{aligned}
\hat{N}_{\mathrm{e}} &= \hat{\rho}_{\mathrm{e}}(0);\\
\hat{N}_{\mathrm{h}} &= \hat{\rho}_{\mathrm{h}}(0);\\
\hat{\rho}(0) &= \hat{N}_{\mathrm{e}} - \hat{N}_{\mathrm{h}};\\
\hat{D}(0) &= \hat{N}_{\mathrm{e}} + \hat{N}_{\mathrm{h}};
\end{aligned}$$

The e–h Fermi operators depend on the two quantum numbers. In the Landau gauge one of them is the wave number p and the second one is the quantum number n of the Landau level. In the LLL approximation n is zero and its notation is dropped. The wave number p enumerates the N-fold degenerate states of the 2D electrons in a strong magnetic field. N can be expressed through the layer surface area S and the magnetic length l as follows: $N = S/2\pi l^2$; $l^2 = \hbar c/eH$, where H is the magnetic field strength.

The above operators determine the structure of the 2D e–h system Hamiltonian in the LLL approximation. In the previous papers [2, 3, 5–7] the initial Hamiltonian was gauge-invariant and has the form

$$\hat{H} = \frac{1}{2}\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}} \left[\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q}) - \hat{N}_{\mathrm{e}} - \hat{N}_{\mathrm{h}}\right] - \mu_{\mathrm{e}}\hat{N}_{\mathrm{e}} - \mu_{\mathrm{h}}\hat{N}_{\mathrm{h}}, \tag{2}$$

where

$$W_{\boldsymbol{Q}} = \frac{2\pi e^2}{\varepsilon_0 S|\boldsymbol{Q}|}\exp\left[-\frac{Q^2 l^2}{2}\right]; \quad \mu = \mu_{\mathrm{e}} + \mu_{\mathrm{h}}. \tag{3}$$

The energy of the 2D magnetoexciton $E_{\mathrm{ex}}(P)$ depends on the 2D wave vector $\boldsymbol{P}$ and forms a band with the dependence

$$\begin{aligned}
E_{\mathrm{ex}}(\boldsymbol{P}) &= -I_{\mathrm{ex}}(\boldsymbol{P}) = -I_l + E(\boldsymbol{P});\\
I_{\mathrm{ex}}(\boldsymbol{P}) &= I_l \mathrm{e}^{-\frac{P^2 l^2}{4}} I_0\left(\frac{P^2 l^2}{4}\right);\\
I_l &= \frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}}; \quad \sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}} = I_l.
\end{aligned} \tag{4}$$

Instead of the chemical potential μ in Eq. (3) we will use the value $\overline{\mu}$ accounted from the bottom of the exciton band

$$\overline{\mu} = \mu - E_{\mathrm{ex}}(0) = \mu + I_l. \tag{5}$$

In the case of BEC of the magnetoexcitons on the state with $\boldsymbol{k} \neq 0$, the chemical potential accounted from the exciton level $E_{\mathrm{ex}}(\boldsymbol{k})$ leads to the expression

$$\mu - E_{\mathrm{ex}}(\boldsymbol{K}) = \overline{\mu} - E(\boldsymbol{K}). \tag{6}$$

To introduce the phenomenon of BEC of excitons the gauge symmetry of the initial Hamiltonian was broken with the help of a unitary transformation, following the KKK method [13]:

$$D(\sqrt{N_{\mathrm{ex}}}) = e^{\hat{X}}; \quad \hat{X} = \sqrt{N_{\mathrm{ex}}}\left(e^{i\varphi}d^{\dagger}(\boldsymbol{k}) - e^{-i\varphi}d(\boldsymbol{k})\right). \tag{7}$$

Instead of the usual procedure introducing the Bogoliubov (u,v) transformation from the Fermi operators of the initial electrons and holes to the Fermi operators of the new quasiparticles we will remain in the frame of the initial Fermi operators. This leads to the Hamiltonian with broken gauge symmetry similar to the Bogoliubov theory of quasiaverages

$$\begin{aligned}
\hat{\tilde{\mathcal{H}}} &= e^{\hat{X}}\hat{H}e^{-\hat{X}} = \hat{H} + \frac{1}{1!}\left[\hat{X},\hat{H}\right] + \frac{1}{2!}\left[\hat{X},\left[\hat{X},\hat{H}\right]\right]\\
&\quad + \frac{1}{3!}\left[\hat{X},\left[\hat{X},\left[\hat{X},\hat{H}\right]\right]\right] + \ldots\\
&= \hat{H} + \sqrt{N_{\mathrm{ex}}}(\overline{\mu} - E(\boldsymbol{k}))\left(e^{i\varphi}d^{\dagger}(\boldsymbol{k}) + e^{-i\varphi}d(\boldsymbol{k})\right) + \hat{\mathcal{H}}'.
\end{aligned} \tag{8}$$

Neglecting the term $\hat{\mathcal{H}}'$ we obtain the starting Hamiltonian in the frame of the Bogoliubov theory of quasiaverages:

$$\begin{aligned}
\hat{\mathcal{H}} &= \frac{1}{2}\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}}\left[\rho(\boldsymbol{Q})\rho(-\boldsymbol{Q}) - \hat{N}_{\mathrm{e}} - \hat{N}_{\mathrm{h}}\right]\\
&\quad - \mu_{\mathrm{e}}\hat{N}_{\mathrm{e}} - \mu_{\mathrm{h}}\hat{N}_{\mathrm{h}} - \eta\sqrt{N}\left(e^{i\varphi}d^{\dagger}(\boldsymbol{k}) + e^{-i\varphi}d(\boldsymbol{k})\right),
\end{aligned} \tag{9}$$

where

$$\eta = (E_{ex}(K) - \mu)v = (E(K) - \overline{\mu})v;$$
$$v = v^2; \quad N_{ex} = v^2 N.$$

The ground-state wave function is the same as in the KKK method

$$\left|\psi_g(\boldsymbol{k})\right\rangle = D\left(\sqrt{N_{ex}}\right)|0\rangle, \tag{10}$$

where $|0\rangle$ is the vacuum state.

The equations of motion for the operators (1) are obtained using the commutation relations. They are

$$\begin{aligned} i\hbar\frac{\mathrm{d}}{\mathrm{d}t}d(\boldsymbol{P}) &= [d(\boldsymbol{P}), \hat{\mathcal{H}}];\\ i\hbar\frac{\mathrm{d}}{\mathrm{d}t}d^\dagger(2\boldsymbol{K}-\boldsymbol{P}) &= [d^\dagger(2\boldsymbol{K}-\boldsymbol{P}), \hat{\mathcal{H}}];\\ i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(\boldsymbol{P}-\boldsymbol{K}) &= [\hat{\rho}(\boldsymbol{P}-\boldsymbol{K}), \hat{\mathcal{H}}];\\ i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\hat{D}(\boldsymbol{P}-\boldsymbol{K}) &= [\hat{D}(\boldsymbol{P}-\boldsymbol{K}), \hat{\mathcal{H}}]. \end{aligned} \tag{11}$$

Following the equations of motion (10) we will introduce four interconnected retarded Green's functions at $T = 0$ [25, 26]

$$\begin{aligned} G_{11}(\boldsymbol{P},t) &= \left\langle\left\langle d(\boldsymbol{P},t); d^\dagger(\boldsymbol{P},0)\right\rangle\right\rangle;\\ G_{12}(\boldsymbol{P},t) &= \left\langle\left\langle d^\dagger(2\boldsymbol{K}-\boldsymbol{P},t); d^\dagger(\boldsymbol{P},0)\right\rangle\right\rangle;\\ G_{13}(\boldsymbol{P},t) &= \left\langle\left\langle \frac{\hat{\rho}(\boldsymbol{P}-\boldsymbol{K},t)}{\sqrt{N}}; d^\dagger(\boldsymbol{P},0)\right\rangle\right\rangle;\\ G_{14}(\boldsymbol{P},t) &= \left\langle\left\langle \frac{\hat{D}(\boldsymbol{P}-\boldsymbol{K},t)}{\sqrt{N}}; d^\dagger(\boldsymbol{P},0)\right\rangle\right\rangle. \end{aligned} \tag{12}$$

Thus, our description is based on the Hamiltonian (9) and the ground-state wave function (10). It corresponds to the coherent state representation with a given value of the chemical potential μ and the number of particles N as a thermodynamic variable. In this representation the operators creating and annihilating the condensate particles are substituted by the macroscopic classical amplitudes and do not participate explicitly in the commutation calculations. Nevertheless, their existence can be observed looking at the distribution of the wave vectors of the operators $d(P)$, $d(2K-P)$, $\rho(P-K)$, and $D(P-K)$. The origin of this distribution becomes clear if the Green's function and their average values are written in another representation with a given number of particles N and with the chemical potential as a thermodynamic variable introduced by Belyaev [27]. In this representation the ground-state wave function is $\left|\phi_H^0(N)\right\rangle$ instead of $\left|\psi_g(\boldsymbol{k})\right\rangle$.

The creation and annihilation operators of the condensed particles must be written in an explicit form and the number of particles must be conserved, otherwise all averages will be equal to zero. The Green's functions must be written in the form with the averages

$$\begin{aligned} &\left\langle\phi_H^0(N)\right|d(P)d^\dagger(P)\left|\phi_H^0(N)\right\rangle;\\ &\left\langle\phi_H^0(N)\right|\frac{d(K)d(K)}{N_{ex}}d^\dagger(2K-P)d^\dagger(P)\left|\phi_H^0(N)\right\rangle;\\ &\left\langle\phi_H^0(N)\right|\frac{d(K)}{\sqrt{N_{ex}}}\frac{D(P-K)}{\sqrt{N}}d^\dagger(P)\left|\phi_H^0(N)\right\rangle;\\ &\left\langle\phi_H^0(N)\right|\frac{d(K)}{\sqrt{N_{ex}}}\frac{\rho(P-K)}{\sqrt{N}}d^\dagger(P)\left|\phi_H^0(N)\right\rangle, \end{aligned} \tag{13}$$

where the summary numbers of condensed and noncondensed excitons is conserved. The same takes place separately for the numbers of electrons and holes because the Fourier transform $D(P-K)$ and $\rho(P-K)$ of the density operators does not change the number of fermions but only their total momenta, which are transferred from the fermion to the exciton subsystem, so that the momentum conservation law is fulfilled in the framework of the whole system.

The Fourier transforms of the Green's functions (11) are denoted as

$$\begin{aligned} G_{11}(\boldsymbol{P},\omega) &= \left\langle\left\langle d(\boldsymbol{P})|d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega;\\ G_{12}(\boldsymbol{P},\omega) &= \left\langle\left\langle d^\dagger(2\boldsymbol{K}-\boldsymbol{P})\right|d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega;\\ G_{13}(\boldsymbol{P},\omega) &= \left\langle\left\langle \frac{\hat{\rho}(\boldsymbol{P}-\boldsymbol{K})}{\sqrt{N}}\,\middle|\, d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega;\\ G_{14}(\boldsymbol{P},\omega) &= \left\langle\left\langle \frac{\hat{D}(\boldsymbol{P}-\boldsymbol{K})}{\sqrt{N}}\,\middle|\, d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega. \end{aligned} \tag{14}$$

The equation of motion for one-operator Green's functions $G_{1j}(\boldsymbol{P},\omega)$, where $j = 1,2,3,4$, give rise to new two-operator (four-particle) Green's functions generated by the nonlinear terms in the equations of motion (11) for the operators (1):

$$\begin{aligned} &\left\langle\left\langle\hat{\rho}(\boldsymbol{Q})d(\boldsymbol{P}-\boldsymbol{Q})|d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega;\\ &\left\langle\left\langle d^\dagger(2\boldsymbol{K}-\boldsymbol{P}-\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\right|d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega;\\ &\left\langle\left\langle \frac{\hat{\rho}(\boldsymbol{P}-\boldsymbol{K}-\boldsymbol{Q})}{\sqrt{N}}\hat{\rho}(\boldsymbol{Q})\,\middle|\, d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega;\\ &\left\langle\left\langle \frac{\hat{D}(\boldsymbol{P}-\boldsymbol{K}-\boldsymbol{Q})}{\sqrt{N}}\hat{\rho}(\boldsymbol{Q})\,\middle|\, d^\dagger(\boldsymbol{P})\right\rangle\right\rangle_\omega. \end{aligned}$$

It is a well-known situation described by Zubarev [26]. For the two-operator Green's functions of the first generation the new equations of motion were obtained. This is the second step in the frame of the method, which will form the second link of an infinite chain of equations of motion. Both links constructed in such a way will be exact in the frame of the Hamiltonian (9). The new equations of motion contain in their components new types of three-operator, six-particle Green's functions of the first generation as well as new types

of the two-operator Green's functions of the second generation, and so on.

As was said above, the three-operator Green's functions of the first generation have appeared, being accompanied by the new two-operator Green's functions of the second generation. Exactly the same evolution of the equations of motion takes place for all four starting Green's functions $G_{1j}(\boldsymbol{P},\omega)$. The truncation of the infinite chains of equations of motion is needed using a reasonable approximation. Following Zubarev's method [26] we will truncate the three-operator Green's functions expressing them through the one-operator Green's functions (14) multiplied by the average values of another two remaining operators. This method, if applied to the two-operator Green's functions of the second generation, in fact, means to their linearization.

The linearization can be achieved conserving only the macroscopic large values of the operators substituting them by their average values at some well-defined values of the wave vector and neglecting all their infinitesimal values as follows

$$\begin{aligned} d(\boldsymbol{P}) &\simeq \delta_{kr}(\boldsymbol{P},\boldsymbol{K})e^{i\varphi}\sqrt{N_{\mathrm{ex}}};\\ d^{\dagger}(\boldsymbol{P}) &\simeq \delta_{kr}(\boldsymbol{P},\boldsymbol{K})e^{-i\varphi}\sqrt{N_{\mathrm{ex}}};\\ \hat{D}(\boldsymbol{P}) &\simeq \delta_{kr}(\boldsymbol{P},0)\langle\hat{D}(0)\rangle \simeq \delta_{kr}(\boldsymbol{P},0)2N_{\mathrm{ex}};\\ \rho(\boldsymbol{P}) &\simeq \delta_{kr}(\boldsymbol{P},0)\langle\hat{\rho}(0)\rangle = 0. \end{aligned} \tag{15}$$

The truncation procedure was successful applied in the case of electron–phonon interaction for the metals in normal states, and also for the superconductors. It can be applied also in the case of Bose–Einstein condensed magnetoexcitons because this phenomenon was taken into account from the very beginning by the Bogoliubov method of quasiaverages. The calculations of the average values of the products of two operators exctracted from the left-hand side of the three-operator Green's functions are performed using the ground-state wave function of the Bose–Einstein condensed magnetoexcitons. In this way supplementary simplifications of the cumbersome expressions are obtained.

The truncations and the decouplings of the three-operator Green's functions generated by all four equations of motion are effectuated using the approximations

$$\begin{aligned} &\left\langle\left\langle \frac{\hat{\rho}(\boldsymbol{P}-\boldsymbol{K}-\boldsymbol{Q}-\boldsymbol{R})}{\sqrt{N}}\hat{\rho}(\boldsymbol{R})\hat{\rho}(\boldsymbol{Q}) \,\middle|\, d^{\dagger}(\boldsymbol{P})\right\rangle\right\rangle_{\omega}\\ &\approx G_{13}(\boldsymbol{P},\omega)[\delta_{kr}(\boldsymbol{Q},\boldsymbol{P}-\boldsymbol{K})\langle\hat{\rho}(\boldsymbol{R})\hat{\rho}(-\boldsymbol{R})\rangle\\ &\quad+(\delta_{kr}(\boldsymbol{R},-\boldsymbol{Q})+\delta_{kr}(\boldsymbol{R},\boldsymbol{P}-\boldsymbol{K}))\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle]; \end{aligned} \tag{16}$$

etc.

Here, the average is calculated using the ground-state wave function $|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle$ (10) of the Bose–Einstein condensed 2D magnetoexcitons as

$$\begin{aligned} &\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle,\ \langle\hat{\rho}(\boldsymbol{Q})\hat{D}(-\boldsymbol{Q})\rangle,\\ &\langle\rho(\boldsymbol{Q}+\boldsymbol{K}-\boldsymbol{P})d(\boldsymbol{P}-\boldsymbol{Q})\rangle\sqrt{N},\\ &\langle d^{\dagger}(2\boldsymbol{K}-\boldsymbol{P}-\boldsymbol{Q})\rho(\boldsymbol{K}-\boldsymbol{P}-\boldsymbol{Q})\rangle\sqrt{N}. \end{aligned}$$

It can be shown that these averages depend essentially and in some cases are proportional to the small parameter of the theory $v^2(1-v^2)$ related to the e–h pairs concentration. After the truncations and linearizations the multioperator Green's functions are expressed through the one-operator Green's function $G_{1j}(\boldsymbol{P},\omega)$, with $j=1,2,3,4$, and their four equations of motion can be written in a closed form introducing the self-energy parts $\Sigma_{ij}(\boldsymbol{P},\omega)$ as follows

$$\sum_{j=1}^{4} G_{1j}(\boldsymbol{P},\omega)\Sigma_{jk}(\boldsymbol{P},\omega) = C_{1k};\quad k=1,2,3,4. \tag{17}$$

There are 16 different components of the self-energy parts forming the 4×4 matrix.

Most of the self-energy parts $\Sigma_{ij}(\boldsymbol{P},\omega)$ contain the average values of the two-operator products. They were calculated using the ground-state wave function $|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle$ and have the expressions

$$\begin{aligned} &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\\ &= 4u^2v^2N\sin^2\left(\frac{[\boldsymbol{K}\times\boldsymbol{Q}]_z l^2}{2}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|\hat{\rho}(\boldsymbol{Q}+\boldsymbol{P}-\boldsymbol{K})\hat{\rho}(\boldsymbol{K}-\boldsymbol{P}-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\\ &= 4u^2v^2N\sin^2\left(\frac{[\boldsymbol{K}\times(\boldsymbol{P}+\boldsymbol{Q})]_z l^2}{2}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|\hat{\rho}(\boldsymbol{Q})\hat{D}(-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\\ &= 2iu^2v^2N\sin\left(\frac{[\boldsymbol{K}\times\boldsymbol{Q}]_z l^2}{1}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|\hat{\rho}(\boldsymbol{Q}+\boldsymbol{P}-\boldsymbol{K})\hat{D}(\boldsymbol{K}-\boldsymbol{P}-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\\ &= 2iu^2v^2N\sin\left(\frac{[\boldsymbol{K}\times(\boldsymbol{P}+\boldsymbol{Q})]_z l^2}{1}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|d^{\dagger}(\boldsymbol{K}-\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\sqrt{N}\\ &= 2iuv^3N\sin\left(\frac{[\boldsymbol{K}\times\boldsymbol{Q}]_z l^2}{2}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|d^{\dagger}(\boldsymbol{P}-\boldsymbol{Q})\hat{\rho}(\boldsymbol{P}-\boldsymbol{Q}-\boldsymbol{K})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\sqrt{N}\\ &= -2iuv^3N\sin\left(\frac{[\boldsymbol{K}\times(\boldsymbol{P}-\boldsymbol{Q})]_z l^2}{2}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|\hat{\rho}(\boldsymbol{Q})d(\boldsymbol{K}-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\sqrt{N}\\ &= -2iuv^3N\sin\left(\frac{[\boldsymbol{K}\times\boldsymbol{Q}]_z l^2}{2}\right);\\ &\langle\psi_{\mathrm{g}}(\boldsymbol{k})|\hat{\rho}(\boldsymbol{P}+\boldsymbol{Q})d(\boldsymbol{K}-\boldsymbol{P}-\boldsymbol{Q})|\psi_{\mathrm{g}}(\boldsymbol{k})\rangle\sqrt{N}\\ &= -2iuv^3N\sin\left(\frac{[\boldsymbol{K}\times(\boldsymbol{P}+\boldsymbol{Q})]_z l^2}{2}\right). \end{aligned} \tag{18}$$

All these averages are proportional to N and essentially depend on the wave vectors and on the small parameters of the types u^2v^2 or uv^3. However, only the averages of the type $\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle$ are real and positive with a constant sign at any values of the wave vectors.

All other averages are purely imaginary changing their signs in dependence on their arguments leading to a small absolute values of the corresponding self-energy parts. All of them will be dropped to simplify the equation of motion for the Green's functions.

In spite of the approximations made concerning the many operator Green's functions and the averages of the two-operator products, the obtained self-energy parts remain quite cumbersome. An important simplification can be achieved by choosing the collinear geometry of the experimental observation of the elementary excitations when their wave vectors $\boldsymbol{P}$ are oriented parallel or have an exactly opposite direction to the condensate wave vector $\boldsymbol{k}$.

3 Elementary excitations of the Bose–Einstein condensed magnetoexcitons in the collinear geometry The dispersion equation is expressed in general form by the determinant equation

$$\det\left|\Sigma_{ij}(\boldsymbol{P},\omega)\right| = 0; \quad \boldsymbol{P} = \boldsymbol{K} + \boldsymbol{q}. \tag{19}$$

It can be considerably simplified in the collinear geometry, when the wave vectors $\boldsymbol{P}$ of the elementary excitations are parallel or antiparallel to the Bose–Einstein condensate wave vector $\boldsymbol{k}$. We represent the wave vectors $\boldsymbol{P}$ in the form $\boldsymbol{P} = \boldsymbol{k} + \boldsymbol{q}$, counting them from the condensate wave vector $\boldsymbol{k}$. The relative wave vector $\boldsymbol{q}$ is also collinear to $\boldsymbol{k}$. In this case the projections of the wave vector products $[\boldsymbol{P}\times\boldsymbol{K}]_z$ as well as all coefficients proportional to $\sin([\boldsymbol{P}\times\boldsymbol{K}]_z l^2/2)$ and a half of the matrix elements $\Sigma_{ij}(\boldsymbol{P},\omega)$ in Eq. (19) vanish. The determinant Eq. (19) splits into two independent equations. One of them concerns only the optical plasmons and has a simple form

$$\Sigma_{33}(\boldsymbol{K}+\boldsymbol{q};\omega) = 0; \quad [\boldsymbol{q}\times\boldsymbol{K}]_z = 0, \tag{20}$$

whereas the second equation contains only the diagonal self-energy parts Σ_{11}, Σ_{22}, Σ_{44} and the quasiaverage theory constant η

$$\begin{aligned}&\Sigma_{11}(\boldsymbol{K}+\boldsymbol{q},\omega)\Sigma_{22}(\boldsymbol{K}+\boldsymbol{q},\omega)\Sigma_{44}(\boldsymbol{K}+\boldsymbol{q},\omega)\\ &-2\eta^2(\Sigma_{11}(\boldsymbol{K}+\boldsymbol{q},\omega)+\Sigma_{22}(\boldsymbol{K}+\boldsymbol{q},\omega)) = 0.\end{aligned} \tag{21}$$

This determines three interconnected branches. Two of them describe the proper collective excitations of Bose–Einstein condensed magnetoexcitons and the third branch concerns the acoustical plasmons. In spite of the collinear condition $[\boldsymbol{q}\times\boldsymbol{K}]_z = 0$, Eqs. (20) and (21) and their energy spectra $\omega(\boldsymbol{q})$ are not invariant under the inversion operation when $\boldsymbol{q}$ is substituted by $-\boldsymbol{q}$, because there is a well-defined direction in the system defined by the wave vector $\boldsymbol{k}$. For this reason the elementary excitations with wave vector $\boldsymbol{q}$ and $-\boldsymbol{q}$ have different energies.

Solutions of the dispersion Eq. (21) will be discussed in two limiting cases. One of them is the point $\boldsymbol{k} = 0$, where the system behaves as an ideal Bose gas and another case of finite magnitude of wave vectors $\boldsymbol{kl} \sim 3-4$, when the Bose–Einstein condensed 2D magnetoexcitons can exist in the form of metastable dielectric liquid phase or of dielectric droplets if the correlation energy is taken into account. The correlation energy will not be included in the calculations below. Therefore, the average value $\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle$ and similar expressions are determined in HFBA by the formula (18). They are characterized by a coherence factor $\sin^2([\boldsymbol{k}\times\boldsymbol{Q}]_z l^2/2)$, which vanishes in the point $\boldsymbol{k} = 0$.

All contributions to the self-energy parts proportional to square of the Coulomb interaction matrix elements W_Q^2 multiplied by the averages $\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle$ also vanish in this limit $\boldsymbol{k}\to 0$. Therefore the 2D magnetoexciton system becomes a pure ideal gas if the influence of the ELLs is neglected. This surprising result was revealed for the first time by Lerner and Lozovik [1–3] and was confirmed by Paquet et al. [5].

The collective elementary excitations are calculated for the conditions $\boldsymbol{kl} \sim 3-4$ and $\nu^2 \approx \nu_m^2$ when the ground state of the magnetoexcitons is similar to the metastable dielectric liquid phase if the correlation energy is taken into account. However, in the present consideration the collective elementary excitations, the self-energy parts and the chemical potential are determined only in the HFBA without correlation corrections.

Even in collinear geometry the diagonal self-energy parts $\Sigma_{ii}(\boldsymbol{K}+\boldsymbol{q},\omega)$ with $i = 1,2,3,4$ and $\boldsymbol{kl} = 3-4$ cannot be calculated analytically for arbitrary values of the relative wave vector $\boldsymbol{q}$. For this reason we will consider the analytical expressions for $\boldsymbol{kl} \approx 3-4$ and $\boldsymbol{ql} \le 1 < \boldsymbol{kl}$ using series expansions of the small values $\boldsymbol{ql} < 1$ compared with $\boldsymbol{kl} \approx 3-4$.

The self-energy parts $\Sigma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega)$ and $\Sigma_{22}(\boldsymbol{k}+\boldsymbol{q},\omega)$ in the collinear geometry are

$$\begin{aligned}\Sigma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega) &= (\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q})+i\delta)\\ &\quad -4\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[(\boldsymbol{k}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\quad\times\frac{\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle}{\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{Q})+i\delta}\\ &= \sigma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega)+i\Gamma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega);\end{aligned} \tag{22}$$

$$\begin{aligned}\Sigma_{22}(\boldsymbol{k}+\boldsymbol{q},\omega) &= (\hbar\omega-\overline{\mu}+E(\boldsymbol{k}-\boldsymbol{q})+i\delta)\\ &\quad -4\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[(\boldsymbol{k}-\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\quad\times\frac{\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle}{\hbar\omega-\overline{\mu}+E(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{Q})+i\delta}\\ &= \sigma_{22}(\boldsymbol{k}+\boldsymbol{q},\omega)+i\Gamma_{22}(\boldsymbol{k}+\boldsymbol{q},\omega).\end{aligned}$$

They were separated in the real and imaginary parts and obey to the equalities

$$\begin{aligned}&\omega)\sigma_{22}(\boldsymbol{k}+\boldsymbol{q},\omega)\\&\quad=-\sigma_{11}(\boldsymbol{k}-\boldsymbol{q},-\omega)\Gamma_{22}(\boldsymbol{k}+\boldsymbol{q},\omega)\\&\quad=\Gamma_{11}(\boldsymbol{k}-\boldsymbol{q},-\omega).\end{aligned}\tag{23}$$

The real and imaginary parts σ_{11} and Γ_{11} are

$$\begin{aligned}&\sigma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega)=\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q})\\&\quad-4\sum_{\boldsymbol{Q}}W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[(\boldsymbol{k}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\&\quad\times\frac{Pf\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle}{\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{Q})};\\&\Gamma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega)=4\pi\sum_{\boldsymbol{Q}}W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[(\boldsymbol{k}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\&\quad\times\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle\delta(\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{Q})).\end{aligned}\tag{24}$$

We take their series expansion on the parameter $ql<1$, which is small compared with $\boldsymbol{kl}\approx 3-4$. At the same time we will expand the denominator $\hbar\omega+\overline{\mu}-E(\boldsymbol{K}+\boldsymbol{q}-\boldsymbol{Q})$ in Eq. (24) taking into account that the most probable value of $\boldsymbol{Ql}$ is unity ($\overline{\boldsymbol{Ql}}\approx 1$) and $\overline{\boldsymbol{Ql}}$ is less than the elementary excitation wave vector $|\boldsymbol{k}+\boldsymbol{q}|l$.

In this approximation, with $\boldsymbol{kl}\approx 3-4$; $ql<1$; and $\overline{\boldsymbol{Ql}}\approx 1$, we can write

$$\frac{1}{\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{Q})}\approx\frac{1}{\hbar\omega+\mu-E(\boldsymbol{k}+\boldsymbol{q})}+\cdots.\tag{25}$$

The presence of the unknown frequency ω in the denominators side by side with another term in numerators leads to the increase of the order of the dispersion equation and of the number of branches in the energy spectrum.

Consider the remaining two diagonal self-energy parts Σ_{33} and Σ_{44}. Their imaginary components are equal to zero. The initial expression is

$$\begin{aligned}&\Sigma_{44}(\boldsymbol{k}+\boldsymbol{q},\omega)=(\hbar\omega+i\delta)\\&-\frac{4}{\hbar\omega+\mathrm{i}\delta}\sum_{\boldsymbol{Q}}W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[\boldsymbol{q}\times\boldsymbol{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle.\end{aligned}\tag{26}$$

The last diagonal self-energy part in $\Sigma_{33}(\boldsymbol{k}+\boldsymbol{q},\omega)$ has the initial form

$$\begin{aligned}&\Sigma_{33}(\boldsymbol{k}+\boldsymbol{q},\omega)=(\hbar\omega+i\delta)\\&-\frac{4}{\hbar\omega+i\delta}\sum_{\boldsymbol{Q}}W_{\boldsymbol{Q}}\sin^2\left(\frac{[\boldsymbol{q}\times\boldsymbol{Q}]_z l^2}{2}\right)\\&\times\{W_{\boldsymbol{Q}}\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle-W_{\boldsymbol{Q}-\boldsymbol{q}}\langle\hat{\rho}(\boldsymbol{q}-\boldsymbol{Q})\hat{\rho}(\boldsymbol{Q}-\boldsymbol{q})\rangle\\&+W_{\boldsymbol{q}}[\langle\hat{\rho}(\boldsymbol{q}-\boldsymbol{Q})\hat{\rho}(\boldsymbol{Q}-\boldsymbol{q})\rangle-\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle]\}.\end{aligned}\tag{27}$$

The self-energy parts in the case of Bose–Einstein condensed magnetoexcitons depend on the average value $\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle$ (18). The expression $\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle$ on the other hand essentially depends on the condensate wave vector $\boldsymbol{k}$ and vanishes at the point $\boldsymbol{k}=0$. As was shown in Ref. [28] in the case of an electron–hole liquid (EHL) the self-energy part $\sigma_{33}(q,\omega)$ describing the intra-LLL excitations has a similar form, but with the average value $\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle=2\nu^2(1-\nu^2)N$.

In what follows it will be convenient to introduce nondimensional variables:

$$\begin{aligned}&\hbar\omega/I_l=\tilde{\omega};\quad\overline{\mu}/I_l=\tilde{\overline{\mu}};\quad\tilde{\eta}=\eta/I_l;\\&\tilde{\sigma}_{ii}=\sigma_{ii}/I_l;\quad\tilde{E}(P)=E(P)/I_l;\\&\tilde{\Gamma}_{ii}=\Gamma_{ii}/I_l;\quad\tilde{I}_{\mathrm{ex}}(\boldsymbol{k})=I_{\mathrm{ex}}(\boldsymbol{k})/I_l;\\&kl=y;\quad ql=x;\quad Pl=z;\\&\cos\alpha=\pm1;\quad\alpha=0,\pi;\quad\boldsymbol{qk}=qk\cos\alpha.\end{aligned}\tag{28}$$

Then, the self-energy parts $\tilde{\sigma}_{11}$ and $\tilde{\sigma}_{22}$ take the form:

$$\begin{aligned}&\tilde{\sigma}_{ii}(x\cos\alpha,y,\tilde{\omega})=u_{ii}(x\cos\alpha,y,\tilde{\omega})\\&\quad+v_{ii}(x\cos\alpha,y,\tilde{\omega})x\cos\alpha+x^2w_{ii}(x\cos\alpha,y,\tilde{\omega}),\\&i=1,2.\end{aligned}\tag{29}$$

The coefficients u_{ii}, v_{ii}, and w_{ii} are

$$\begin{aligned}&u_{11}(x\cos\alpha,y,\tilde{\omega})=\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(y+x\cos\alpha)\\&\quad-\frac{16u^2v^2C_1(y)}{\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(y+x\cos\alpha)};\\&v_{11}(x\cos\alpha,y,\tilde{\omega})=-\frac{16u^2v^2L_1(y)}{\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(y+x\cos\alpha)};\\&w_{11}(x,y,\tilde{\omega})=-\frac{16u^2v^2P_1(y)}{\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(y+x\cos\alpha)};\\&u_{22}(x\cos\alpha,y,\tilde{\omega})=-u_{11}(-x\cos\alpha,y,-\tilde{\omega});\\&v_{22}(x\cos\alpha,y,\tilde{\omega})=v_{11}(-x\cos\alpha,y,-\tilde{\omega});\\&w_{22}(x\cos\alpha,y,\tilde{\omega})=-\mathrm{w}_{11}(-x\cos\alpha,y,-\tilde{\omega}).\end{aligned}\tag{30}$$

The dimensionless chemical potential $\tilde{\mu}$ and quasiaverage constant $\tilde{\eta}$ equal to:

$$\begin{aligned}\tilde{\overline{\mu}} &= \tilde{E}(y)(1-2v^2);\\ \tilde{\eta} &= (\tilde{E}(y)-\tilde{\overline{\mu}})v = 2v^3\tilde{E}(y).\end{aligned} \tag{31}$$

The dispersion relation $\tilde{E}(y)$ for the magnetoexciton is approximated as

$$\tilde{E}(y) = \frac{y^2}{4+y^2}; \quad \tilde{I}_{\mathrm{ex}}(y) = \frac{4}{4+y^2}; \tag{32}$$

$$\frac{\partial\tilde{E}(y)}{\partial y} = \frac{8y}{(4+y^2)^2}; \quad \frac{\partial^2\tilde{E}(y)}{\partial y^2} = \frac{8(4-3y^2)}{(4+y^2)^3}.$$

The imaginary parts of the self-energy parts Σ_{11} and Σ_{22} are

$$\begin{aligned}\Gamma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega) &= 4\pi\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}}^2 \sin^2\left(\frac{[(\boldsymbol{k}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\times \langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle\delta(\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{Q}))\\ &= 16\pi u^2v^2\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}}^2 N\sin^2\left(\frac{[\boldsymbol{K}\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\times \sin^2\left(\frac{[(\boldsymbol{K}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\times \delta(\hbar\omega+\overline{\mu}-E(\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{Q}))\\ &= \Gamma_{22}(\boldsymbol{k}-\boldsymbol{q},-\omega).\end{aligned} \tag{33}$$

The final expression for σ_{44} takes the form:

$$\sigma_{44}(x,y,\tilde{\omega}) = \tilde{\omega} - \frac{x^2}{\tilde{\omega}}\left[u^2v^2\frac{2}{\pi}A_1(y)\right], \tag{34}$$

and the simplified expression for $\sigma_{33}(\boldsymbol{k}+\boldsymbol{q},\omega)$ is

$$\begin{aligned}\sigma_{33}(x,y,\omega) &= \hbar\omega - \frac{4}{\hbar\omega}\sum_{\boldsymbol{Q}} W_{\boldsymbol{Q}}\left(W_{\boldsymbol{Q}}-W_{\boldsymbol{Q}-q}\right)\\ &\times \sin^2\left(\frac{[\boldsymbol{q}\times\boldsymbol{Q}]_z l^2}{2}\right)\langle\hat{\rho}(\boldsymbol{Q})\hat{\rho}(-\boldsymbol{Q})\rangle.\end{aligned} \tag{35}$$

The coefficients in these expressions were determined in Ref. [29]. The first of them is $C_1(y)$:

$$\begin{aligned}C_1(y) &= \frac{1}{8\pi}\left\{E_i(-y^2) - 4E_i\left(-\frac{y^2}{4}\right)\right.\\ &\left.+ 3\ln(y^2) + 3C - 8\ln 2\right\}\\ &= \frac{1}{8\pi}\left\{\sum_{k=1}^{\infty}\frac{(-y^2)^k}{k(k!)} - 4\sum_{k=1}^{\infty}\frac{\left(-\frac{y^2}{4}\right)^k}{k(k!)}\right\},\end{aligned} \tag{36}$$

where $C = 0.577216$ is the Euler constant, $E_i(y)$ is the integral exponential function [30].

The second coefficient $L_1(y)$ is:

$$\begin{aligned}L_1(y) = \frac{1}{4\pi}y[&\exp\left(-\frac{y^2}{4}\right){}_iF_1\left(1;2;\frac{y^2}{4}\right)\\ &- \exp\left(-y^2\right){}_iF_1(1;2;y^2)].\end{aligned} \tag{37}$$

The next coefficient is:

$$\begin{aligned}P_1(y) = \frac{1}{16\pi}\left\{-\frac{1}{2}+\exp\left(-\frac{y^2}{4}\right)\right.\\ -\frac{1}{2}\exp\left(-y^2\right)+\frac{y^2}{4}\left[\exp\left(-y^2\right){}_iF_1(2;3;y^2)\right.\\ \left.\left.-\frac{1}{2}\exp\left(-\frac{y^2}{4}\right){}_iF_1\left(2;3;\frac{y^2}{4}\right)\right]\right\}.\end{aligned} \tag{38}$$

The perturbation theory used here is based on a small parameter $v^2(1-v^2)$ reflecting the filling factor of the LLLs and the concentration of the e–h system. The Hamiltonian is written and the calculations are made for the given chemical potential μ representation with the particle numbers being a thermodynamic variable. Representation in the form of Eq. (13) with a given number of particles N was used to better understand the involved physical processes. The starting Hamiltonian (9) contains explicitly operators $d^{\dagger}(\boldsymbol{k})$ and $d(\boldsymbol{k})$ describing the creation and annihilation of the condensed excitons, but the following equations of motion for the noncondensed particles do not contain these operators.

A small parameter of the perturbation theory is introduced in the calculations by the average values $\langle\hat{\rho}(Q)\hat{\rho}(-Q)\rangle$. The presence of the BEC in the system is determined by the parameters μ and η, which also depend on the filling factor $v = v^2$.

The self-energy parts $\sigma_{ii}(P,\omega)$ in the zero-order approximation do not contain the average values $\langle\hat{\rho}(Q)\hat{\rho}(-Q)\rangle$ and the acoustical plasmon branch with frequencies equal to zero does not exist. In this approximation there are two exciton branches of the energy spectrum shown in Fig. 2. One of them with positive energy is the energy branch, and the second curve with negative values is the quasienergy branch. Another quasienergy branch can be obtained from the energy branches by means of their twice successive reflections, one of them as regards the x-axis, which change the sign of the energy and the next reflection as regards the y-axis, which transpose the picture from the x to the $-x$ direction.

We shall discuss only the origin of the energy branches. The exciton energy branch in Fig. 2 depends on the wave vector $\boldsymbol{P} = \boldsymbol{k}+\boldsymbol{q}$ in the laboratory reference frame. The branch is influenced by the motion of the condensate with the group velocity $\boldsymbol{V}_{\mathrm{g}}(\boldsymbol{k})$ in this reference frame. This leads to the dispersion law with linear dependence in the range of small wave vectors $\boldsymbol{q}$. The slope as well as the group velocity decrease with the increase of the condensate wave vector $\boldsymbol{k}$

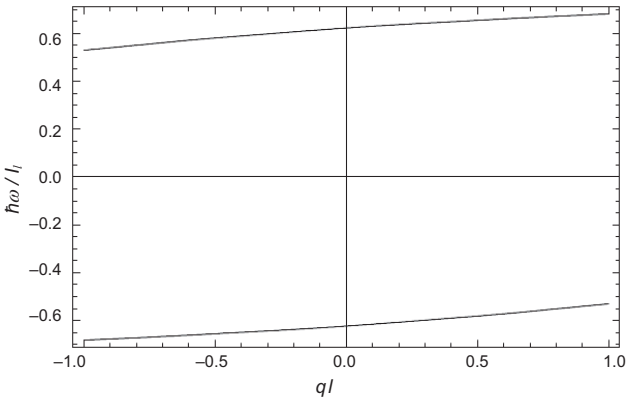

Figure 2 The energy spectrum of the elementary excitations of the magnetoexcitons in the case when the self-energy parts $\sigma_{ii}(P,\omega)$ were taken into account in the zero-order approximation. The dimensionless wave vector of the BEC-ed magnetoexcitons is 3.6.

because in the range $y = \boldsymbol{kl} \geq 3-4$ the dispersion law of the free magnetoexciton has a saturation dependence. In the first-order approximation the terms proportional to the averages $\langle\hat{\rho}(Q)\hat{\rho}(-Q)\rangle$ included in the compositions of the self-energy parts $\sigma_{ii}(P,\omega)$ were taken into account.

An additional simplification of the self-energy parts σ_{11} and σ_{22} was used extracting the denominators containing the unknown frequency ω from the summation procedure according to the formula (25). These denominators double the number of solutions of the dispersion equation. The obtained results are shown in Fig. 3. There are three energy branches. One of them is a gapless branch. It describes the dispersion law of the acoustical plasmons arising in the e–h system in the condition of BEC of 2D magnetoexcitons with wave vectors $y = \boldsymbol{kl} \geq 3-4$. This dispersion law depends on the relative wave vector $\boldsymbol{q}$ counted from the condensate wave vector $\boldsymbol{k}$. The energy spectrum describes the acoustical plasmons in the reference frame

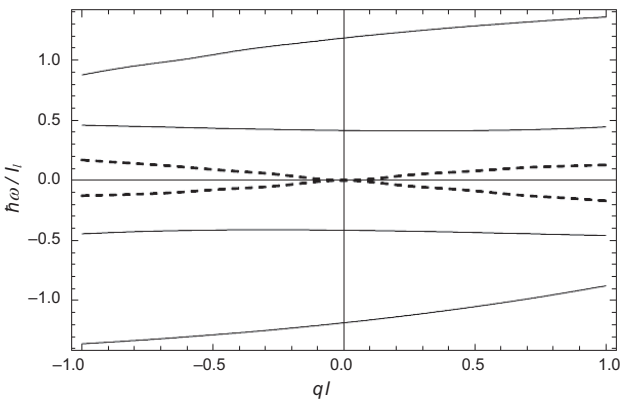

Figure 3 The energy spectrum of the mixed exciton–plasmon (solid line) and acoustical plasmon (dashed lines) collective elementary excitations of the Bose–Einstein condensed 2D magnetoexcitons with dimensionless wave vector $y = 3.6$ and filling factor $\nu^2 = 0.28$.

moving together with the condensate. For this reason it is not affected by the group velocity of the condensate as it happened with the exciton energy spectrum described in the laboratory reference frame. The dimensionless energy of the acoustical plasmons in the range of the small values of $x = \boldsymbol{ql}$ at arbitrary values of $y = \boldsymbol{kl}$ equals

$$\tilde{\omega}_{\mathrm{ap}}(x,y) = uvx\left(\frac{2}{\pi}A_1(y)\right)^{1/2}; \quad x<1; \quad x = ql; \quad y = kl, \tag{39}$$

where the coefficient $A_1(y)$ is

$$A_1(y) = \frac{1}{2} - \frac{1}{2}\mathrm{e}^{-y^2/4} + \frac{y^2}{16}\mathrm{e}^{-y^2/4}{}_1F_1\left(1,3,y^2/4\right), \quad A_1(y)\underset{y\to 0}{=}\frac{3}{16}y^2; \quad A_1(y)\underset{y\to\infty}{=}\frac{1}{2}. \tag{40}$$

The acoustical plasmons do not exist at the point $\boldsymbol{k} = 0$ because the 2D magnetoexcitons with $\boldsymbol{k} = 0$ form an ideal Bose gas as pointed out by Lerner and Lozovik [1–3] and confirmed by Paquet et al. [5]. This is true if the influence of the ELLs is neglected. The BEC of 2D magnetoexcitons with wave vector $\boldsymbol{k} = 0$ under the influence of ELLs was considered in Ref. [8].

The appearance of the second gapped energy branch in addition to the zero-order exciton energy branch is due to the presence of the Bose–Einstein condensate generating many-particle complexes composed of the condensed excitons and noncondensed excitons or plasmons. Two such complexes were introduced in the formula (13) being represented by the operators

$$\frac{d(\boldsymbol{k})}{\sqrt{N_{\mathrm{ex}}}}\frac{D(\boldsymbol{P}-\boldsymbol{k})}{\sqrt{N}} \quad \text{and} \quad \frac{d(\boldsymbol{k})d(\boldsymbol{k})}{N_{\mathrm{ex}}}d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P}).$$

Both of them have the resulting wave vector $\boldsymbol{P} = \boldsymbol{k} + \boldsymbol{q}$, as the noncondensed exciton represented by the operator $d(\boldsymbol{P})$. Their time evolution is described in the laboratory reference frame. We believe that the second gapped energy branch reflects the existence of a compound complex formed by a condensed exciton with the wave vector $\boldsymbol{k}$ and by a plasmon arising in its surroundings with the wave vector $\boldsymbol{q}$ in the moving reference frame. The energy of the complex is greater than the energy of the acoustical plasmon with wave vector $\boldsymbol{q}$ by the energy of the condensed exciton $(-\mu + E(\boldsymbol{k}))$. The summary energy is comparable with the energy of the acoustical plasmon with wave vector $\boldsymbol{k} + \boldsymbol{q}$ in the laboratory reference frame. The interplay of the complex condensed-exciton–plasmon and the noncondensed exciton gives rise to another two superposition states of the polariton-type with two branches the compositions of which depend on the wave vector $\boldsymbol{P}$. The existence of the superposition states steams from the equations of motion for the operators $d(\boldsymbol{P})$, $d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P})$ and $D(\boldsymbol{P}-\boldsymbol{k})/\sqrt{N}$ given by formulas (4). In their right-hand sides in the collinear geometry there are terms linear in the operators $d(\boldsymbol{P}), d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P})$ and

$D(\boldsymbol{P}-\boldsymbol{k})/\sqrt{N}$ with the coefficients proportional to η, indicating that only in the presence of the Bose–Einstein condensate are such linear combinations possible. This statement is confirmed also by the Dyson Eq. (17), which contains the linear combination of three Green's functions $G_{11}(P,\omega)$, $G_{12}(P,\omega)$, and $G_{14}(P,\omega)$. In the collinear geometry we have

$$\begin{aligned} &G_{11}(P,\omega)\sigma_{11}(P,\omega)-\eta G_{14}(P,\omega)=C_1;\\ &G_{12}(P,\omega)\sigma_{22}(P,\omega)+\eta G_{14}(P,\omega)=C_2;\\ &\quad-2\eta(G_{11}(P,\omega)-G_{12}(P,\omega))\\ &\quad+G_{44}(P,\omega)G_{14}(P,\omega)=C_4. \end{aligned} \tag{41}$$

The plasmon-type Green's function $G_{14}(P,\omega)$ enters into linear combination with the difference between the normal and abnormal exciton Green's functions $(G_{11}(P,\omega)-G_{12}(P,\omega))$, and this superposition state exists only if the constant η introducing the BEC of 2D magnetoexcitons in the Bogoliubov theory of quasiaverages is not zero. The abnormal exciton Green's functions are not different from zero only in the conditions of BEC.

The self-energy parts $\sigma_{11}(P,\omega)$ and $\sigma_{22}(P,\omega)$ are interconnected by the symmetry relations (23). The structure of each of them, for example, of $\sigma_{11}(P,\omega)$, contains the terms with exciton and plasmon origins. It is evident in the approximation (25) when the self-energy part $\sigma_{11}(P,\omega)$ has the form

$$\begin{aligned} \sigma_{11}(\boldsymbol{k}+\boldsymbol{q},\omega)&=\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(\boldsymbol{k}+\boldsymbol{q})\\ &\quad-\frac{T^2(\boldsymbol{k}+\boldsymbol{q})}{\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(\boldsymbol{k}+\boldsymbol{q})}, \end{aligned} \tag{42}$$

where

$$\begin{aligned} T^2(\boldsymbol{k}+\boldsymbol{q})&=4\sum_{\boldsymbol{Q}}W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[(\boldsymbol{k}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\quad\times\langle\rho(\boldsymbol{Q})\rho(-\boldsymbol{Q})\rangle; \end{aligned}$$

$$\begin{aligned} \tilde{T}(\boldsymbol{k}+\boldsymbol{q})&=\tilde{T}(y,x\cos\alpha)\\ &=\frac{2}{I_l}\Big(\sum_{\boldsymbol{Q}}W_{\boldsymbol{Q}}^2\sin^2\left(\frac{[(\boldsymbol{k}+\boldsymbol{q})\times\boldsymbol{Q}]_z l^2}{2}\right)\\ &\quad\times\langle\rho(\boldsymbol{Q})\rho(-\boldsymbol{Q})\rangle\Big)^{1/2}\\ &=4u\mathrm{v}\sqrt{C_1(y)+x\cos\alpha L_1(y)+x^2P_1(y)}. \end{aligned} \tag{43}$$

At the point $x=0$ we have $T(y,0)=2u\mathrm{v}(C_1(y))^{1/2}$, whereas in the limit $y=\boldsymbol{kl}\to 0$ and $x<1$ we obtain

$$\begin{aligned} C_1(y)&=\frac{3}{16\pi}y^4;\\ L_1(y)&=\frac{3}{16\pi}y^3;\\ P_1(y)&=\frac{3}{128\pi}y^2;\\ \tilde{T}_{\substack{y\to 0\\ x<1}}&=uvxy\left(\frac{3}{8\pi}\right)^{1/2}. \end{aligned} \tag{44}$$

It is seen that the energy related to the exciton component in the structure of $\sigma_{11}(P,\omega)$ is $\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(\boldsymbol{k}+\boldsymbol{q})$, whereas the energy related to the plasmon formation is represented by the expression $\tilde{T}(y,x\cos\alpha)$. The complete expression (42) for $\sigma_{11}(P,\omega)$ consists of two parts with their relative contributions determined by the relation $T^2(\boldsymbol{k}+\boldsymbol{q})/(\tilde{\omega}+\tilde{\overline{\mu}}-\tilde{E}(\boldsymbol{k}+\boldsymbol{q}))$.

All these arguments allow us to find the existence of new superposition state between the noncondensed exciton and the compound complex composed by the condensed exciton and the plasmon in a moving reference frame. The existence of the mixed exciton–plasmon collective elementary excitations was established in Ref. [29]. The present paper presents for the first time some additional details and a different point of view.

The authors of Ref. [31] pointed that the spinor BEC is an ideal system to study the physics of the quasi-Nambu–Goldstone (NG) modes because such a system has a great experimental manipulability and well-established microscopic Hamiltonian. It was shown in Ref. [31] that the quasi-NG modes appears in a spin-2 nematic phase. In the nematic condensate, three phases, each of which has a different symmetry, are energetically degenerate to zeroth order [32] and the zeroth-order solution has a rotational symmetry SO(5), whereas the Hamiltonian of the spin-2 condensate has a rotational symmetry SO(3). The ground-state symmetry of the nematic phase at zeroth-order approximation is broken by quantum corrections, thereby making the quasi-NG modes massive. The breaking of the SO(5) symmetry occurs. The number n of the quasi-NG modes was determined by Georgi and Pais [33]. In Ref. [31] it was presented as

$$n=\dim(\tilde{M})-\dim(M), \tag{45}$$

where $\tilde{M}$ is the surface on which the effective potential reaches its minimal value in the zeroth order and $\dim(\tilde{M})$ is the dimension of this surface. The dimension $\dim(M)$ determines the number of the NG modes. This implies [31] that M is a submanifold of $\tilde{M}$ and n is the dimension of the complementary space of M inside $\tilde{M}$.

Returning to the case of 2D magnetoexcitons in the BEC state with nonzero wave vector $\boldsymbol{k}$ described by the

Hamiltonian (9), one should recall that it lost the continuous symmetries that existed in the initial form (2). It happened due to the presence of the term $\eta\left(d^{\dagger}(\boldsymbol{k})+d(\boldsymbol{k})\right)$ in the Bogoliubov theory of quasiaverages. Nevertheless, the energy of the ground state as well as the self-energy parts $\Sigma_{ij}(P,\omega)$, which determine the energy spectrum of the collective elementary excitations, depend only on the modulus of the wave vector $\boldsymbol{k}$ and do not depend on its direction. All these expressions have a rotational symmetry SO(2), in spite of the fact that the symmetry of the Hamiltonian (9) was lost. We believe that this is the condition described by Georgi and Pais [33] favoring the emergence of the quasi-NG modes. We explain the existence of the gapped, massive exciton-type branches of the collective elementary excitations obtained in our calculations by these considerations.

4 Conclusions In the present paper the origin and conditions of appearance of the mixed-type exciton–plasmon collective elementary excitations revealed in Ref. [29] are discussed in detail. The main condition is the existence of the Bose–Einstein condensed 2D magnetoexcitons with moving dipole moments giving rise to the compound complexes formed by the condensed excitons and noncondensed excitons or plasmons. They are described by the operators for the given values of N

$$\frac{d(\boldsymbol{k})}{\sqrt{N_{\mathrm{ex}}}}\,\frac{D(\boldsymbol{P}-\boldsymbol{k})}{\sqrt{N}} \text{ and } \frac{d(\boldsymbol{k})d(\boldsymbol{k})}{N_{\mathrm{ex}}}d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P}),$$

or for the given μ by the operators

$$\frac{D(\boldsymbol{P}-\boldsymbol{k})}{\sqrt{N}} \text{ and } d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P}).$$

The first complex can be denoted as a condensed-exciton–plasmon complex, and the second one as a noncondensed-exciton quasienergy complex. The interplay of these complexes with the noncondensed excitons described by the operator $d(\boldsymbol{P})$ is stimulated by the condensate that acts on the whole e–h system similar to an external macroscopic field. This action is determined by the constant η introducing the gauge symmetry breaking in the Bogoliubov quasiaverage theory manner. The superposition of these three bare states leads to the formation of the new-type collective elementary excitations. Both complexes and the noncondensed exciton have the same wave vector $\boldsymbol{P}=\boldsymbol{k}+\boldsymbol{q}$ in the given N representation. On the contrary, in the given μ representation the condensed excitons operators $d(\boldsymbol{k})$ and $d^{\dagger}(\boldsymbol{k})$ are present only in the Hamiltonian (9), but they are absent in the equations of motion (11) for the operators $d(\boldsymbol{P})$, $d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P})$, and $D(\boldsymbol{P}-\boldsymbol{k})/\sqrt{N}$ as well as in the Dyson equations (17) for the corresponding Green's functions being substituted by a constant $\sqrt{N_{\mathrm{ex}}}=\mathrm{v}\sqrt{\mathrm{N}}$.

The condensed-exciton–plasmon complex has the total energy equal to the energy $(-\overline{\mu}+E(\boldsymbol{k}))$ necessary to create condensed excitons moving with the wave vector $\boldsymbol{k}$ and group velocity $\boldsymbol{V}_{\mathrm{g}}(\boldsymbol{k})$ in the laboratory reference frame, and to the energy $\hbar\omega_{\mathrm{ap}}(\boldsymbol{q})$ necessary to create an acoustical plasmon in the surroundings of the condensed exciton, *i.e.*, in the moving reference frame. These two components of the complex have total wave vector $\boldsymbol{P}=\boldsymbol{k}+\boldsymbol{q}$ and the energy comparable with the acoustical plasmon energy in the laboratory reference frame, $\hbar\omega_{\mathrm{ac}}(\boldsymbol{k}+\boldsymbol{q})=T(\boldsymbol{k}+\boldsymbol{q})$. The substitution of the complex total energy by the value $T(\boldsymbol{k}+\boldsymbol{q})$ took place when the approximation (25) was adopted by extracting the denominator with unknown frequency $\tilde{\omega}$ from the summation procedure.

The equations of motion (11) and the Dyson equations (17) show that the superposition takes place between the states represented by the operator $D(\boldsymbol{P}-\boldsymbol{k})/\sqrt{N}$ and the difference $(d(\boldsymbol{P})-d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P}))$ in the given μ representation or in a given N representation between the complex states described by the operators

$$\frac{d(\boldsymbol{k})}{\sqrt{N_{\mathrm{ex}}}}\,\frac{D(\boldsymbol{P}-\boldsymbol{k})}{\sqrt{N}} \text{ and } \left(d(\boldsymbol{P})-\frac{d(\boldsymbol{k})d(\boldsymbol{k})}{N_{\mathrm{ex}}}d^{\dagger}(2\boldsymbol{k}-\boldsymbol{P})\right).$$

Since the interaction constant is equal to η such superposition is not possible in the absence of BEC. The same conclusion can be made looking at the Dyson equations (41), where the acoustical plasmon Green's function G_{14} is interconnected with the difference of the exciton normal and abnormal Green's functions $(G_{11}(P,\omega)-G_{12}(P,\omega))$.

The superposition between the exciton state related with the creation or the annihilation of an e–h pair and of a plasmon state taking place with the conservation of the particle number cannot be realized without the presence of the BEC and of the participation of the condensed exciton forming the complex with a plasmon. On the contrary, in the presence of BEC such a superposition is possible.

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