COLLECTIVE ELEMENTARY EXCITATIONS OF BOSE-EINSTEIN CONDENSED TWO-DIMENSIONAL MAGNETOEXCITONS STRONGLY INTERACTING WITH ELECTRON-HOLE PLASMA

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Abstract

The collective elementary excitations of a system of Bose-Einstein condensed two-dimensional magnetoexcitons interacting with electron-hole(e-h) plasma in a strong perpendicular magnetic field are studied. The breaking of the gauge symmetry is introduced into the Hamiltonian following the Bogoliubov`s theory of quasiaverages.

The motion equations for the summary operators describing the creation and annihilation of magnetoexcitons as well as the density fluctuations of the electron-hole(e-h) plasma were derived. They suggest the existence of magneto-exciton-plasmon complexes, the energies of which differ by the energies of one or two plasmon quanta.

Starting with these motion equations one can study the Bose-Einstein Condensation (BEC) of different magneto-exciton-plasmon complexes introducing different constants of the broken symmetry correlated with their energies. The Green`s functions constructed from these summary operators are two-particle Green`s functions. They obey the chains of equations expressing the two-particle Green`s functions through the four-particle and six-particle Green`s functions. These chains were truncated in such a way that the six-particle Green`s functions, were expressed through the two-particle ones. At the same time the elementary excitations with different wave vectors were decoupled. As a result of these simplifications the Dyson-type equation in a matrix form for the two-particle Green`s functions was obtained.

The 4×4 determinant constructed from the self-energy part $\sum_{ij}(\vec{P},\omega)$ gives rise to dispersion equation. The dispersion relations were obtained in analytical form, when in the self-energy parts $\sum_{ij}(\vec{P},\omega)$ only the terms linear in Coulomb interaction were kept. Taking into account also the terms quadratic in Coulomb interaction the dispersion equation becomes cumbersome and it can be solved only numerically.

1.Introduction.

In previous papers [1-5] the coherent pairing of two-dimensional electrons and holes in a strong perpendicular magnetic field was studied. In last papers [4,5] it was shown, that the Bose-Einstein Condensation (BEC) of magnetoexcitons with different from zero wave vector $\vec{k}$ and motional dipole moments essentially differs from the case $k=0$. The supplementary attraction between the parallel aligned in-plane motional dipole moments gives rise to the metastable dielectric liquid phase. Its chemical potential reaches the minimal value at some filling factor of the lowest Landau level (LLL) and lies on the energy scale below or in the vicinity of the chemical potential of the degenerate Bose gas of magnetoexcitons with $k=0$. In these conditions the drops of the dielectric liquid phase are surrounded by the degenerate

Bose gas and the coexistence of two BEC-tes is possible. The correlation energy due to coherent excited states of BEC-ed magnetoexcitons becomes important at significant values of wave vectors and vanishes in the point $k = 0$. On the contrary the influence of the excited Landau levels is especially efficient on the BECed magnetoexcitons with the wave vector $k = 0$ and rapidly decreases with the increasing of k. In difference on the chemical potential the collective elementary excitations of the BEC-ed magnetoexcitons practically were not studied. Some preliminary remarks were made in [3]. This question happens to be unusual and our paper completely is devoted to it. We realized that in two-dimensional electron-hole system in a strong perpendicular magnetic field the role of plasmon oscillations is similar with the role of magnetic flux quanta in the case of 2D electron gas in the condition of the fractional quantum Hall effect (FQHE) [6]. The magnetic flux quanta induce the vortices formation in the electron gas. The electron being accompanied by a few vortices forms a composite particle of a fermion or boson types.

One can also remember the case of electron gas in the field of laser radiation. The electron state accompanied by a photon gives rise to quasi – energy states [7]. Returning to the case of collective elementary excitations in the system of BEC-ed two-dimensional magnetoexcitons we must remark that they are inseparable from the plasma oscillations. They are strongly interconnected and must be considered simultaneously. The same happens with the exciton gas interacting with phonons in deformable lattices. But there are some more unusual properties. The motion equations for the exciton creation and annihilation operators as well for the density fluctuation operators, as we will see below contain free terms and terms describing the nonlinearity in the system due to the Coulomb interaction in the two-dimensional e-h system. The dispersion relation for the free excitons looks as $E_{ex}(\vec{P}) = -I_l + E(\vec{P})$ where I_l is the ionization potential of the magnetoexciton with two-dimensional wave vector $\vec{P} = 0$ and $E(\vec{P})$ is the proper dispersion relation, which changes quadratically in the range of small wave vectors $\vec{P}$ and tends to the finite value I_l when P tends to infinite, so as $E_{ex}(P)$ tend to zero.

The free energy for the plasmon looks as $E(\vec{P})$ and coincide exactly with the second term in the dispersion relation of magneto-exciton. The magneto-exciton with wave vector $\vec{P}$ can be regarded as magnetoexciton with wave vector $\vec{P} = 0$ and a plasmon with wave vector $\vec{P}$. The magnetoexciton can be regarded as a simple quasiparticle and at the same time as a complex consisting of an exciton and a plasmon, when the part depending on the wave vector $\vec{P}$ is determined by the plasmon. Such interpretation follows from the properties of the motion equations. They will be analyzed in detail below. In contrast to the 2D e-h gas the 3D electron plasma has plasmon oscillations with a energy gap [8], whereas the collective elementary excitations of a 3D Bose-gas have gapless energy spectrum. By this reason the interconnection of the exciton and plasmon elementary excitation in 3D system does not appear.

Below we will study this interconnection in 2D-e-h system in a strong magnetic field in detail. But for the beginning a short review of the papers dedicated to the study of the collective elementary excitations in the system of 2D two-component electron-electron and electron-hole gases is presented.

As one remember [8] the plasma oscillations in three-dimensional (3D) crystals are determined by the frequency ω_p satisfying the relation

$$\omega_p^2 = \frac{4\pi e^2 n_v}{\varepsilon_o m}; \qquad n_v = \frac{N_e}{V} \tag{1}$$

Where N_e is the number of electrons, V is the volume of the crystal, n_e is the bulk electron density, m is the effective electron mass and ε_0 is the dielectric constant of the crystal. In the two-dimensional ideal monolayer with the surface area S in the similar way one can derive the dispersion relation

$$\omega_p^2(q) = \frac{2\pi e^2 n_s q}{\varepsilon_0 m}; \qquad \omega_p(q) \sim \sqrt{q}; \qquad n_s = \frac{N_e}{S}; \tag{2}$$

Here n_S is the surface electron density. The differentce between two expressions (1) and (2) is due to the change of the role of Coulomb interaction in two different dimensionalities. Two Fourier transforms V_K of the Coulomb potential and the kinetic energy T_K of the electron have the forms

$$V_K^{3D} = \frac{4\pi e^2}{V\varepsilon_0 k^2}; \qquad V_K^{2D} = \frac{2\pi e^2}{\varepsilon_0 S k}; \qquad T_K = \frac{\hbar^2 k^2}{2m} \tag{3}$$

It is recognized in literature that the role of Coulomb interaction is enhanced in 2D structure in comparison with the bulk crystals, whereas the kinetic energy remains with the same quadratic dependence on wave vector $\vec{k}$ in the absence of the strong magnetic field.
The both expressions (1) and (2) can be join by a formula

$$\hbar^2 \omega_p^2(k) = 2 N_e T_K V_K \tag{4}$$

Das Sarma and Madhukar [9] have investigated theoretically the longitudinal collective modes of spatially separated two-component two-dimensional plasma in solids using the generalized random phase approximation. It can be realized in semiconductor heterojunctions and superlattices. The two-layer structure with two-component plasma is discussed below. It has long been known that two-component plasma has two branches to its longitudinal oscillations. The higher frequency branch is named as optical plasmon (OP). Here the two carrier densities of the same signs oscillate in-phase and their density fluctuation operators $\hat{\rho}_{e,1}(\vec{Q})$ and $\hat{\rho}_{e,2}(\vec{Q})$ form an in-phase superposition

$$\hat{\rho}_{OP}(\vec{Q}) = \hat{\rho}_{e,1}(\vec{Q}) + \hat{\rho}_{e,2}(\vec{Q}) \tag{5}$$

In the case of opposite signs electron and hole charges they oscillate out-of-phase and their charge density fluctuation operators $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$ combine in out-of-phase manner

$$\hat{\rho}_{OP}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q}) \tag{6}$$

The lower frequency branch is named as acoustical plasmon (AP). Now the carriers of different signs oscillate in-phase, whereas the carriers of the same signs oscillate out-of-phase. Their charge density fluctuation operators combine in the form

$$\hat{\rho}_{AP}(\vec{Q}) = \hat{\rho}_{e,1}(\vec{Q}) - \hat{\rho}_{e,2}(\vec{Q}); \qquad \hat{\rho}_{AP}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q}) \tag{7}$$

The optical and acoustical branches have the dispersion relations in the long wavelength region as follows

$$\omega_{OP}(q) \sim \sqrt{q}; \qquad \omega_{AP}(q) \sim q; \qquad q \to 0 \tag{8}$$

By virtue of spatial separation z between the two components of the 2D plasma the AP branch becomes with a greater slope of the linear q dependence, because this slope is proportional to z, when z is of the order of Bohr radius a_B. At small $z \to 0$ the AP branch lies inside the single-particle excitation spectrum of the faster moving charged carriers. They leave the corresponding Fermi seas crossing the Fermi energies of the degenerate Fermi gases. This single-particle spectrum is severely Landau damped. At larges values $z > a_B$ the Coulomb interaction between charges in different layers can be neglected and each layer supports an ordinary 2D plasma oscillations with a dispersion $\omega_P(q) \sim q^{1/2}$ [9].

They have been thoroughly studied experimentally In the absence of a magnetic field, but have not so far been addressed in the presence of an external magnetic field. The transformation of the optical and acoustical plasma excitations under the influence of an external perpendicular to the layer magnetic field was studied experimentally by the authors of paper [10], using the AlGaAs/GaAs double quantum well (DQW) and magnetic fields up till 10T.

As was mentioned in this paper in a perpendicular magnetic field many –body interactions become relevant as the electron kinetic energy is completely quenched and the strong Coulomb interaction drives the two-dimensional electron system (2DES) into new phases of matter such as incompressible fractional quantum Hall liquid or Wigner crystal.

In paper [10] the acoustical and optical plasmons at zero field were investigated first. The dispersion relation of the AP was measured in the whole range of accessible in-plane momenta and it was found to be a nearly linear dependence in agreement with the theory.

The entire H field range covered was cut into two parts. In one range the influence of the Bernstein modes (BM) on to the principal plasmons AP and OP can be neglected. The Bernstein modes are charge-density magnetoexcitons having energies $n\hbar\omega_c$, $n \ge 2$ at $ql \to 0$, where ω_c is the cyclotron frequency and l is the magnetic length

$$\omega_c = \frac{eH}{mc}; \qquad\qquad l^2 = \frac{\hbar c}{eH} \tag{9}$$

In another range of magnetic field the AP_S and OP_S resonate with BM_S.

When the BM_S can be neglected the energies of the principal plasmons are monotonically increasing functions of H field, slowly covering to the cyclotron energy. In the limit $H \to 0$ the both plasmon excitations can be approximated

$$\omega^2(q \mid H \ne 0) = \omega^2(q \mid H = 0) + \omega_c^2 \tag{10}$$

The complex anticrossing behavior close to the resonances between AP and OP with BM_S was observed.

The plasmon oscillations in one-component system on the monolayer in a strong perpendicular magnetic field were studied by Girvin, MacDonald and Platzman [11], who proposed the magnetoroton theory of collective excitations in the conditions of the fractional quantum Hall effect (FQHE). The FQHE occurs in low-disorder, high-mobility samples with partially filled Landau levels with filling factor of the form $\nu = \frac{1}{m}$, where m is an integer, for which there is no single-particle gap. In this case the excitation is a collective effect arising from many-body correlations due to the Coulomb interaction. Considerable progress has recently been achieved toward understanding the nature of the many-body ground state well described by Laughlin variational wave function [12]. The theory of the collective excitation spectrum proposed by [11] is closely analogous to Feynman`s theory of superfluid helium [13]. The main Feynman`s arguments lead to the conclusions that on general grounds the low lying excitations of any system will include density waves. As regards the 2D system the perpendicular magnetic field quenches the single particle continuum of kinetic energy leaving a series of discrete highly degenerate Landau levels spaced in energy at intervals $\hbar\omega_c$. In the case of filled Landau level $\nu = 1$ because of Pauli exclusion principle the lowest excitation is necessarily the cyclotron mode in which particles are excited into the next Landau level. In the case of FQHE the lowest Landau level (LLL) is fractionally filled. The Pauli principle no longer excludes low-energy intra-Landau-level excitations. For the FQHE case the low-lying excitations have the primary importance rather than the high-energy inter-Landau-level cyclotron modes [11]. The spectrum has a relatively large excitation gap at zero wave vector $kl = 0$ and in addition it exhibits a deep magneto-roton minimum at $kl \sim 1$ quite analogous to the roton minimum in helium. The magneto-roton minimum becomes deeper and deeper at the decreasing of the filling factor ν in the row $1/3, 1/5, 1/7$ and is the precursor to the gap collapse associated with the Wigner crystallization which occurs at $\nu = \frac{1}{7}$. For larges wave vectors the low lying mode crosses over from being a density wave to becoming a quasiparticle excitation [11]. The Wigner crystal transition occurs slightly before the roton mode goes completely soft. The magnitude of the primitive reciprocal lattice vector for the crystal lies close to the position of the magneto-roton minimum. The authors of [11] suggested also the possibility of pairing of two rotons of opposite momenta leading to the bound two-roton state with small total momentum, as it is known to occur in helium. In contrast to the case of fractional filling factor, the excitations from a filled Landau level in the 2DEG were studied by Kallin and Halperin [14]. They considered an interacting two-dimensional electron system with a uniform positive background in a strong perpendicular magnetic field at zero temperatures. It was supposed that an integral number of Landau levels is filled and the Coulomb energy $\frac{e^2}{\varepsilon_0 l}$ is smaller than the cyclotron energy $\hbar\omega_c$.

The elementary neutral excitations may be described alternatively as magnetoplasma modes or as magnetoexcitons formed by a hole in a filled Landau level and an electron in an empty level. In contrast to the hole in the valence band, which takes part in the formation of the usual magnetoexciton, we deal with the hole in the conduction band, namely in its filled Landau level. It can be denoted as (c,n,h). Its bound state with the electron in the empty Landau level with number n' in the same conduction band denoted as (c, n', e) gives rise to the magnetoexciton named as integer quantum Hall exciton. It is characterized by a conserved wave vector $\vec{k}$ in Landau gauge. The dispersion relation may be calculated exactly to first order in $I_l / \hbar\omega_c$, where I_l is the ionization potential of magnetoexciton with $k = 0$ and equals to $\frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}}$.

The lowest magnetoplasmon band comes in to the cyclotron frequency $m\hbar\omega_c$ at $k = 0$, where $m = n' - n$, if the Coulomb electron-electron interaction is neglected. If the Coulomb interaction is included, then the energy of neutral plasmon will come to the value $m\hbar\omega_c - I_l$. Excitation modes with $m = 0$ do not exist if the initial state has an integer occupation numbers of the Landau levels of both spins. In the ferromagnetic ground state the $m = 0$ excitations are spin waves.

Apal'kov and Rashba[15] considered a case of an electron-hole pair in the presence of an incompressible liquid formed by electrons in the condition of the fractional quantum Hall effect (FQHE). The magnetoplasmons have a dispersion law similar to the rotons in liquid helium and are named as magnerotons. They play the role of phonons in the incompressible liquid and influence on the state of exciton interacting with plasmons. This influence is analogous with the influence of the phonons on the states of electrons or excitons interacting with crystal lattice oscillations in bulk semiconductors and is named as polaron effect. The authors of [15] arrived to the conclusion that the influence of magnetorotons, leads to a giant suppression of the magnetoexciton dispersion in symmetric case. There is a region in the momentum space, where the elementary excitations are interpreted as bound states of a phonon (magnetoroton) with a slow magnetoexciton. As was mentioned the interaction of the exciton with the fluid can be treated as a polaron effect resulting from a dressing by magnetorotons. The polaron shift is zero at $k=0$ in symmetric systems. When the confinement planes for electrons and hole have a distance z different from zero (asymmetric case), the polaron shift of the exciton level is positive, what is determined in an asymmetric system by the influence of the Pauli exclusion principle which is not compensated by the ordinary polaron effect.

Fertig [16] investigated the excitation spectrum of two-layer and three-layer electron systems. In particular case the two-layer system in a strong perpendicular magnetic field with filling factor $\nu=\frac{1}{2}$ of the lowest Landau level (LLL) in the conduction band of each layer was considered. Inter-layer separation z was introduced. The spontaneous coherence of two-component two-dimensional (2D) electron gas was introduced constructing the function

$$\left|\Psi\right\rangle=\prod_k (na_k^+ + \mathrm{v}b_k^+)\left|0\right\rangle \quad ; \quad u^2=\mathrm{v}^2=1/2 \quad , \tag{11}$$

where a_k^+, a_k are the creation and annihilation operators of spin polarized electrons on the LLL of the layer **a** and b_k^+, b_k play the same role for the electrons resided on the layer **b**.

Here the vacuum state $\left|0\right\rangle$ was introduced

$$a_k\left|0\right\rangle=b_k\left|0\right\rangle=0 \tag{12}$$

Both half filled layers **a** and **b** are accompanied by a substrate with positive charge guaranteeing the electrical neutrality of the system. The half filled layer **a** can be considered as a full filled with electrons in the LLL of the conduction band and a half filled by holes in the LLL of the same conduction band. The wave function of the full filled LLL of the layer **a** can be written as

$$\left|\Psi_0\right\rangle=\prod_k a_k^+\left|0\right\rangle \tag{13}$$

The hole creation operator in the conduction band of the layer **a** can be introduced

$$d_k^+=a_{-k} \tag{14}$$

The electrons of the full filled conduction band are compensated by the charge of the substrate and we can only consider the electrons on the layer **b** and the holes on the layer **a**.

Then the wave function (11) of the coherent two-layer electron system can be rewritten in the form

$$\left|\Psi\right\rangle=\prod_k (u+\mathrm{v}b_k^+ d_{-k}^+)\left|\Psi_0\right\rangle \quad , \tag{15}$$

which coincides with the BCS-type wave function of the superconductor. It represents the coherent pairing of the conduction electrons on the LLL of the layer **b** with the holes in the LLL of the conduction band of the layer **a** and describes the BEC of such unusual excitons named as FQHE excitons, because they appear in the conditions proper to the observation of the fractional quantum Hall effect. Here only the BEC on the single exciton state with wave vector $\vec{k} = 0$ is considered.

Fertig has determined the energy spectrum of the elementary excitations in the frame of this ground state. In the case of z=0 the lowest-lying excitations of the system are the higher energy excitons.

Because of the neutral nature of the $\vec{k} = 0$ excitons the dispersion relation of these excitations is given in a good approximation by

$$\hbar\omega(k) = E_{ex}(k) - E_{ex}(0), \tag{16}$$

where $E_{ex}(k)$ is the energy of exciton with wave vector $\vec{k}$. This result was first obtained by Paquet, Rice and Ueda [3,17] using a random phase approximation [RPA]. In the case z=0 the dispersion relation $\omega(k)$ vanishes as k^2 for $k \to 0$, as one expect for Goldstone modes.

For z>0 $\omega(k)$ behaves as an acoustical mode $\omega(k) \sim k$ in the range of small k, whereas in the limit $k \to \infty$ $\omega(k)$ tends to the ionization potential $\Delta(z)$ in the form

$$\hbar\omega(k) = \Delta(z) - \frac{e^2}{\varepsilon_0 k l^2} \tag{17}$$

In the region of intermediate values of k, when $kl \sim 1$, the dispersion relation develops the dips as z is increased. At certain critical value of z=z_{er} the modes in the vicinity of the minima become equal to zero and are named as soft modes. Their appearance testifies that the two-layer system undergo a phase transition to the Wigner crystal state.

The similar results concerning the linear and quadratic dependences of the dispersion relations in the range of small wave vectors q were obtained by Kuramoto and Horie [18], who studied the coherent pairing of electrons and holes spatially separated by the insulator barrier in the structure of the type coupled double quantum wells (CDQW).

The magnetic field is sufficiently strong, so that the carriers populate only their lowest Landau levels (LLL) in the conduction and valence bands. Apparently the electron-hole interaction becomes less important than the repulsive electron-electron and hole-hole interactions as the separation d increases. However at low densities the ground state of the system will be the excitonic phase, instead of the Wigner lattice, for which the repulsive interaction is responsible. The reason is that the energy per electron-hole pair in the excitonic phase is lower than in Wigner crystal. The BEC of magnetoexcitons in the state with zero total momentum was considered and the dispersion relation of the collective excitation modes was derived. In the case $d \neq 0$ the lowest excitation branch has a linear dispersion relation in the region of small wave vectors q $\omega(q) \sim ql$; whereas at d=0 it transforms in the quadratic dependence $\omega(q) \sim (ql)^2$; Kuramoto and Horie mentioned that the linear dispersion relation originates in the fact that at $d \neq 0$ the repulsive Coulomb interaction prevails and the carriers feel this resulting repulsive long-range force [18]. As in the Bogoliubov theory of weakly interacting Bose gas the repulsive interaction leads to the transformation of the quadratic dispersion relation into another one with the linear dependence at small wave vectors.

Spontaneous Coherence in a two-component electron gas created in bilayer quantum well structure in a strong perpendicular magnetic field was recently studied experimentally by Eisenstein [19] and theoretically by MacDonald [20].

The bilayer electron-electron system is much easy to realize in experiment than e-h bilayer, when the holes are created in the valence band and are spatially separated from the electrons in the conduction band. The experimental indications of spontaneous coherence have been seen first in e-e bilayer, which is analogous to Josephson junction. When the two

2D electron layers each at half-filling of the lowest Landau level (LLL) are sufficiently close together, then the ground state of the system possesses interlayer phase coherence. The ground state can be considered as an equilibrium Bose-Einstein condensate of excitons formed by the electrons on the LLL in the conduction band with the residence on one layer and the holes on the LLL of the conduction band with the residence on another layer. This collective state exhibits the quantum Hall effect when electrical currents are driven in parallel through two layers [19]. Counterflow transport experiments were realized. The oppositely directed currents were driven through the two layers. The counterflow proceeds via the collective transport of neutral particles, i.e. interlayer excitons. The Hall resistance of the individual layer vanishes at $T \to 0$ in the collective phase. A weak dissipation is present at finite temperatures. The free vortices are present at all temperatures being induced by the disorders. The existence of the anticipated Goldstone mode linearly dispersing was confirmed experimentally [19]. This mode is the consequence of a spontaneously broken $U(1)$ symmetry in the bilayer system. Measurements of the tunneling conductance between the layers have shown that the tunneling conductance at zero bias grows explosively, when the separation between the layers is brought below a critical value [19].

The counterflow conductivity and inter-layer tunneling experiments both suggest that the system does not have long range order because of the presence of the unbound vortices nucleated by disorder. The finite phase coherence length appears [20].

The appearance of the soft modes in the spectrum of the collective elementary excitations may signalize not only about the possible phase transition of the two-layer system to the Wigner crystal state or to the charge-density-wave (CDW) of a 2D electron system, but also to another variant of the excitonic charge-density-wave (ECDW) state. This new state was revealed theoretically by Chen and Quinn [21,22], who studied the ground state and the collective elementary excitations of a system consisting of spatially separated electron and hole layers in strong magnetic field. When the interlayer Coulomb attraction in strong electrons and holes pair together to form excitons. Excitonically condensed state of e-h pairs is the preferable ground state. If the layer separation is larger than a critical value, a novel excitonic-density-wave state is found to have a lower energy than either a homogeneous exciton fluid or a double charge-density-wave state in 2D electron system.

All these details and information will permit to better understand the results of our paper, which is organized as follows.

In section two the breaking of the gauge symmetry of the initial Hamiltonian is introduced by an alternative method following the idea proposed by Bogoliubov in his theory of quasiaverages [23]. The equivalence with another Bogoliubov u-v transformation method is revealed.

In section three the motion equations for the operators were obtained, whereas in section four on their base the main equations determining the many-particle Green`s functions were deduced.

Section five is devoted to the discussion of the used approximations. One of them corresponds to the Hartree-Fock-Bogoliubov approximation (HFBA) and the second one to the calculations of the correlation energy [4,5]. The energy spectrum in HFBA is represented in section six. The more complete results are discussed in the seventh section.

2. The breaking of the gauge symmetry of the initial Hamiltonian. Two equivalent representations

For the very beginning we will introduce the operators describing the magneto-excitons and plasmons, and their commutation relations.

The creation and annihilation operators of magnetoexcitons are two-particle operators reflecting the electron-hole (e-h) structure of the excitons. They are denoted below as $d^{\dagger}(\vec{p})$ and $d(\vec{p})$, where $\vec{p}(p_x, p_y)$ is the two-dimensional wave vector. There are also the density fluctuation operators for electrons $\hat{\rho}_e(\vec{Q})$ and for holes $\hat{\rho}_h(\vec{Q})$ as well as their linear combinations $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$. They are determined below

$$
\begin{aligned}
&\hat{\rho}_e(\vec{Q}) = \sum_t e^{iQ_y t l^2} a^{\dagger}_{t-\frac{Q_x}{2}} a_{t+\frac{Q_x}{2}};\\
&\hat{\rho}_h(\vec{Q}) = \sum_t e^{iQ_y t l^2} b^{\dagger}_{t+\frac{Q_x}{2}} b_{t-\frac{Q_x}{2}};\\
&\hat{\rho}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q});\\
&\hat{D}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q});\\
&d^{\dagger}(\vec{P}) = \frac{1}{\sqrt{N}} \sum_t e^{-iP_y t l^2} a^{\dagger}_{t+\frac{P_x}{2}} b^{\dagger}_{-t+\frac{P_x}{2}};\\
&d(\vec{P}) = \frac{1}{\sqrt{N}} \sum_t e^{iP_y t l^2} b_{-t+\frac{P_x}{2}} a_{t+\frac{P_x}{2}};\\
&\hat{N}_e = \hat{\rho}_e(0);\\
&\hat{N}_h = \hat{\rho}_h(0);\\
&\hat{\rho}(0) = \hat{N}_e - \hat{N}_h;\\
&\hat{D}(0) = \hat{N}_e + \hat{N}_h;
\end{aligned}
\tag{18}
$$

and are expressed through the Fermi creation and annihilation operators $a^{\dagger}_p, a_p$ for electrons and $b^{\dagger}_p, b_p$ for holes. The e-h Fermi operators depend on two quantum numbers. In Landau gauge one of them is the wave number p and the second one is the quantum number n of the Landau level. In the lowest Landau level (LLL) approximation n has only the value zero and its notation is dropped. The wave number p enumerates the N-fold degenerate states of the 2D electrons in a strong magnetic field. N can be expressed through the layer surface area S and the magnetic length l as follows

$$
N = \frac{S}{2\pi l^2}; \quad l^2 = \frac{\hbar c}{eH} \quad , \tag{19}
$$

where H is the magnetic field strength. The operators (18) obey the following commutation relations, most of which being for the first time established by Apal`kov and Rashba [15]

$$\left[\hat{\rho}(\vec{Q}),\hat{\rho}(\vec{P})\right]=-2iSin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}(\vec{P}+\vec{Q})$$

$$\left[\hat{D}(\vec{Q}),\hat{D}(\vec{P})\right]=-2iSin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}(\vec{P}+\vec{Q}) \qquad (20)$$

$$\left[\hat{\rho}(\vec{Q}),\hat{D}(\vec{P})\right]=-2iSin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{D}(\vec{P}+\vec{Q})$$

$$\left[d(p),d^+(Q)\right]=\delta_{kr}(\vec{P},\vec{Q})-$$

$$-\frac{1}{N}\left[i\sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}\left(\vec{P}-\vec{Q}\right)+\cos\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{D}\left(\vec{P}-\vec{Q}\right)\right]$$

$$\left[\hat{\rho}\left(\vec{P}\right),d\left(\vec{Q}\right)\right]=2i\sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)d\left(\vec{P}+\vec{Q}\right)$$

$$\left[\hat{\rho}\left(\vec{P}\right),d^+\left(\vec{Q}\right)\right]=-2i\sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)d^+\left(-\vec{P}+\vec{Q}\right)$$

$$\left[\hat{D}\left(\vec{P}\right),d^+\left(\vec{Q}\right)\right]=2\cos\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)d^+\left(\vec{Q}-\vec{P}\right)$$

$$\left[\hat{D}\left(\vec{P}\right),d\left(\vec{Q}\right)\right]=-2\cos\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)d\left(\vec{P}+\vec{Q}\right)$$

One can observe that the density fluctuation operators (18) with different wave vectors $\vec{P}$ and $\vec{Q}$ do not commute. Their non-commutativity is related with the vorticity which accompanies the presence of the strong magnetic field and depends on the vector-product of two wave vectors $\vec{P}$ and $\vec{Q}$ and its projection on the direction of the magnetic field $[\vec{P}\times\vec{Q}]_z$. These properties considerably influence the structure of the motion equations for the operators (1) and determine new aspect of the magneto-exciton-plasmon physics. Indeed in the case of 3D e-h plasma in the absence of the external magnetic field the density fluctuation operators do commute [8]. The magneto-exciton creation and annihilation operators $d^\dagger(\vec{p})$ and $d(\vec{Q})$ as in general case do not obey exactly the Bose commutation rule. Their deviation from it is proportional to the density fluctuation operators $\hat{\rho}(\vec{P}-\vec{Q})$ and $\hat{D}(\vec{P}-\vec{Q})$. The discussed above operators determine the structure of the 2D e-h system Hamiltonian in the LLL approximation. In previous papers [1,2,3,4,5] the initial Hamiltonian was gauge-invariant.

It has the form

$$\hat{H}=\frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}\left[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})-\hat{N}_e-\hat{N}_h\right]-\mu_e\hat{N}_e-\mu_h\hat{N}_h \ , \tag{21}$$

where

$$W_{\vec{Q}}=\frac{2\pi e^2}{\varepsilon_0 S\left|\vec{Q}\right|}Exp\left[-\frac{Q^2l^2}{2}\right]; \quad \mu=\mu_e+\mu_h \tag{22}$$

The energy of the two-dimensional magnetoexciton $E_{ex}(P)$ depends on the two-dimensional wave vector $\vec{P}$ and forms a band with the dependence

$$E_{ex}(\vec{P})=-I_{ex}(\vec{P})=-I_l+E(\vec{P});$$
$$I_{ex}(\vec{P})=I_l e^{-\frac{P^2l^2}{2}} I_0(\frac{P^2l^2}{2}); \qquad I_l=\frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}}; \tag{23}$$

The ionization potential $I_{ex}(P)$ is expressed through the modified Bessel function $I_0(z)$, which has the limiting expressions [10].

$$\underset{z\to 0}{I_0(z)}=1+\frac{z^2}{4}+...; \ \underset{z\to\infty}{I_0(z)}=\frac{e^z}{\sqrt{2\pi z}} \tag{24}$$

It means that the function $E(P)$ can be approximated as follows

$$\underset{P\to 0}{E(\vec{P})}=\frac{\hbar^2P^2}{2M}; \qquad M=M(0)=2\sqrt{\frac{2}{\pi}}\frac{\hbar^2\varepsilon_0}{e^2l};$$
$$\underset{P\to\infty}{E(P)}=I_l(1-\frac{\sqrt{\frac{2}{\pi}}}{Pl}); \quad l^2=\frac{\hbar c}{eH}; \tag{25}$$

Instead of the chemical potential μ (22) we will use the value $\bar{\mu}$ accounted from the bottom of the exciton band

$$\bar{\mu}=\mu-E_{ex}(0)=\mu+I_l; \tag{26}$$

In the case of BEC of the magnetoexcitons on the state with $k\neq 0$ the chemical potential accounted from the exciton level $E_{ex}(k)$ will lead to the expression

$$\mu-E_{ex}(\vec{K})=\bar{\mu}-E(\vec{K}); \tag{27}$$

For introduction of the phenomenon of Bose-Einstein condensation (BEC) of excitons the gauge symmetry of the initial Hamiltonian was broken by the help of the unitary transformation $\hat{D}(\sqrt{N_{ex}})$ following the Keldysh-Kozlov-Kopaev method [24]. We can shortly remember the main outlines of the Keldysh-Kozlov-Kopaev method [24], [25] as it was realized in papers [4,5]. The unitary transformation $\hat{D}(\sqrt{N_{ex}})$ was determined by the formula (25) [4]. Here N_{ex} is the number of condensed excitons. It transforms the operators a_p, b_p to another ones α_p, β_p, as is shown in formulas (30), (31) [4], and gives rise to the BCS-type wave function $\left|\psi_g(\bar{k})\right\rangle$ of the new coherent macroscopic state represented by expression (27) [4]. These results are summarized below

$$\begin{aligned}
&\hat{D}(\sqrt{N_{ex}}) = \exp[\sqrt{N_{ex}}(d^{\dagger}(\bar{k}) - d(\bar{k}))] \\
&\left|\psi_g(\bar{k})\right\rangle = \hat{D}(\sqrt{N_{ex}})\left|0\right\rangle \\
&\alpha_p = \hat{D}a_p\hat{D}^{\dagger} = ua_p - \mathrm{v}(p - \frac{k_x}{2})b^{\dagger}_{k_x - p} \\
&\beta_p = \hat{D}b_p\hat{D}^{\dagger} = ub_p + \mathrm{v}(\frac{k_x}{2} - p)a^{\dagger}_{k_x - p} \\
&a_p = u\alpha_p + \mathrm{v}(p - \frac{k_x}{2})\beta^{\dagger}_{k_x - p} \\
&b_p = u\beta_p - \mathrm{v}(\frac{k_x}{2} - p)\alpha^{\dagger}_{k_x - p}
\end{aligned} \tag{28}$$

$$\begin{gathered}
a_p\left|0\right\rangle = b_p\left|0\right\rangle = 0; \qquad \alpha_p\left|\psi_g(\bar{k})\right\rangle = \beta_p\left|\psi_g(\bar{k})\right\rangle = 0 \\
u = \cos g; \ \ \mathrm{v} = \sin g; \qquad \mathrm{v}(t) = \mathrm{v}e^{-ik_y t l^2} \\
g = \sqrt{2\pi l^2 n_{ex}}; \ \ n_{ex} = \frac{N_{ex}}{S} = \frac{\mathrm{v}^2}{2\pi l^2} \quad g = \mathrm{v}; \ \ \mathrm{v}=\mathrm{Sinv};
\end{gathered} \tag{29}$$

The developed theory [4,5] is true in the limit $\mathrm{v}^2 \approx Sin^2\mathrm{v}$, what means the restriction $\mathrm{v}^2 < 1$. In the frame of this approach the collective elementary excitations can be studied constructing the Green's functions on the base of operators α_p, β_p and dealing with the transformed cumbersome Hamiltonian $\hat{\hat{\mathcal{H}}} = D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$.

We propose another way, which is supplementary but completely equivalent to the previous one and is based on the idea suggested by Bogoliubov in his theory of quasiaverages [23]. Considering the case of a 3D ideal Bose gas with the Hamiltonian

$$H = \sum_{\vec{p}}(\frac{\hbar^2 p^2}{2m} - \mu)a^{\dagger}_{\vec{p}}a_{\vec{p}} \quad , \tag{30}$$

where a_p^+, a_p are Bose operators and μ is the chemical potential, Bogoliubov added a term

$$-\nu\sqrt{V}(a_0 e^{i\varphi} + a_0 e^{-i\varphi}) \tag{31}$$

breaking the gauge symmetry and proposed to consider the BEC on the state with p=0 in the frame of the Hamiltonian

$$\hat{\mathcal{H}} = \sum_p \left(\frac{\hbar^2 p^2}{2m} - \mu\right)a^{\dagger}_p a_p - \nu\sqrt{V}(a^{\dagger}_0 e^{i\varphi} + a_0 e^{-i\varphi}) \quad , \tag{32}$$

where

$$\nu = -\mu\sqrt{\frac{N_0}{V}} = -\mu\sqrt{n_0}; \qquad -\frac{\nu}{\mu} = \sqrt{n_0}; \tag{33}$$

We will name the Hamiltonian of the type (32) as the Hamiltonian of the theory of quasiaverages. It is written in the frame of the operators a_p^+, a_p of the initial Hamiltonian (30).

Our intention is to apply this idea to the case of BEC of interacting 2D magnetoexcitons and to deduce explicitly the Hamiltonian of the type (32) with the finite parameters μ and ν

but with the relation of the type (33). We intend to formulate the new Hamiltonian with broken symmetry in the terms of the operators a_p, b_p avoiding the obligatory crossing to the operators α_p, β_p (28) at least at some stages of the investigation where the representation in the a_p, b_p operators remains preferential.

Of course the two representations are completely equivalent and complimentary each other. We will follow the quasiaverage variant (32) instead of u,v variant (29), because it opens some new possibilities, which were not studied up till now to the best of our knowledge. For example the Hamiltonian of the type (32) is simpler than the Hamiltonian $\hat{\hat{\mathcal{H}}} = D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$ in the α_p, β_p representation and the deduction of the motion equation for the operators (18) and for the many-particle Green's functions constructed on their base is also much simple. We will profit by this advantage at some stages of investigation. On the contrary, when we will deal with the calculations of the average values of different operators on the base of the ground coherent macroscopic state (28) or using the coherent excited states, as we have done in papers [4,5], the most convenient way is to use the α_p, β_p representation. We will use in the wide manner the both representations. The new variant in the style of the theory of quasiaverages can be realized rewriting the transformed Hamiltonian $D(\sqrt{N_{ex}})\hat{H}D^{\dagger}(\sqrt{N_{ex}})$ in the a_p, b_p representation as follows below. To demonstrate it we will represent the unitary transformation

$$\begin{aligned} &\hat{D}(\sqrt{N_{ex}}) = e^{\hat{X}} = \sum_{n=0}^{\infty} \frac{\hat{X}^n}{n!}; \\ &D^{\dagger}(\sqrt{N_{ex}}) = e^{-\hat{X}} \end{aligned} \quad , \tag{34}$$

where

$$\begin{aligned} &\hat{X} = \sqrt{N_{ex}}(e^{i\varphi} d^{\dagger}(K) - e^{-i\varphi} d(K)); \\ &\hat{X}^{\dagger} = -\hat{X}; \end{aligned} \tag{35}$$

The creation and annihilation operators $d^{+}(k), d(k)$ (18) are written in the Landau gauge when the electrons and holes forming the magnetoexcitons are situated on their lowest Landau levels (LLL). Only this variant is considered here without taking into account the excited Landau levels (ELL) , as it was done in [4]. The BEC of 2D magnetoexcitons is considered on the single-exciton state characterized by two-dimensional wave vector $\vec{k}$. Expanding in series the unitary operators $D(\sqrt{N_{ex}}), D^{\dagger}(\sqrt{N_{ex}})$ we will find the transformed operator $\hat{\hat{\mathcal{H}}}$ in the form

$$\hat{\hat{\mathcal{H}}} = e^{\hat{X}} \hat{H} e^{-\hat{X}} = \hat{H} + \frac{1}{1!}\left[\hat{X}, \hat{H}\right] + \frac{1}{2!}\left[\hat{X}, \left[\hat{X}, \hat{H}\right]\right] + \frac{1}{3!}\left[\hat{X}, \left[\hat{X}, \left[\hat{X}, \hat{H}\right]\right]\right] + \ldots = \hat{\mathcal{H}} + \hat{\mathcal{H}}' \tag{36}$$

Here the Hamiltonian $\hat{\mathcal{H}}$ contains the main contributions of the first three terms in the series expansion (36), whereas the operator $\hat{\mathcal{H}}'$ gathers the all remaining terms.

As one can see looking at formulas (35) operator $\hat{X}$ is proportional to the square root of the exciton concentration $\sqrt{N_{ex}}$, which is proportional to the filling number v. One can see that the contributions arising from the first commutator $\left[\hat{X}, \hat{H}\right]$ are proportional to v, the contributions arising from the second commutator $\left[\hat{X}, \left[\hat{X}, \hat{H}\right]\right]$ are proportional to v^2 and so on. Following the Bogoliubov's theory of quasiaverages only the linear terms of the type $\left(d^{+}(k)e^{i\varphi}+e^{-i\varphi}d(k)\right)\nu$ arising from the first commutator $\left[\hat{X},\hat{H}\right]$ must be included into $\hat{\mathcal{H}}$. But taking into account the deviation of the exciton creation and annihilation operators from the pure Bose statistics, we will take into account also the term proportional to $N_{ex}D(0)$ from the second commutator.

We will show below, that such foresight permits to obtain a Hamiltonian $\hat{\mathcal{H}}$ which will generate the motion equation of the exciton creation and annihilation operators in concordance with the basic suppositions concerning their BEC. Such supplimentary term in the Hamiltonian $\hat{\mathcal{H}}$ introduces the needed corrections related with the deviation of the exciton operators from the true Bose statistics. The commutations were effectuated using the commutation relations (20).

The Hamiltonian $\hat{\mathcal{H}}$ with the broken gauge symmetry describing the BEC of 2D magnetoexcitons on the state with wave vector $k\neq 0$ being written in the style of the Bogoliubov's theory of quasiaverages has the form

$$\hat{\mathcal{H}}=\hat{H}+\sqrt{N_{ex}}(\bar{\mu}-E(\vec{K}))(e^{i\varphi}d^{\dagger}(\vec{K})+e^{-i\varphi}d(\vec{K}))+N_{ex}(E(\vec{K})-\bar{\mu})(1-\frac{\hat{D}(0)}{N}); \tag{37}$$

$$\hat{D}(0)=\hat{N}_e+\hat{N}_h;$$

For the case of an ideal 2D Bose gas we can rewrite the coefficient $-\nu\sqrt{V}$ in the Hamiltonian (32), in the form $-\nu\sqrt{N}$ and comparing it with the deduced expression (37), we will find

$$\nu=\left(E(k)-\bar{\mu}\right)\mathrm{v}, \tag{38}$$

where N and the filling number v are determined by expressions (19) and (23). Relation (38) coincides exactly with relation (33) of the Bogoliubov's theory of quasiaverages. In the case of ideal Bose gas ν and $\left(E(k)-\bar{\mu}\right)$ both tend to zero, whereas the filling number is real and different from zero. In the interacting exciton gas the parameter ν and $\left(E(k)-\bar{\mu}\right)$ are both different from zero. But we kept in the expression for $\hat{\mathcal{H}}$ else the last term proportional to $N_{ex}\left(E(k)-\bar{\mu}\right)=N\mathrm{v}^2\left(E(k)-\bar{\mu}\right)$, which was absent in the theory of quasiaverages for the ideal Bose gas. It reflects, as was mentioned above, the deviation of the exciton creation and annihilation operators from the true Bose operators. The influence of the last term will be discussed below writing the motion equations for the exciton operators.

Now the remaining terms gathered in $\hat{\mathcal{H}}'$ will be written. They contain the contributions proportional to $\mathrm{v}^2,\mathrm{v}^3$ and so on. There is also one term proportional to v, but it is nonlinear containing the products of the exciton and plasmon operators. We suppose that their influence on the BEC of magnetoexcitons is less in comparison with the second term in expression (37). The first terms included in $\hat{\mathcal{H}}'$ are

$$\hat{\mathcal{H}}'=-(2i)\sqrt{N_{ex}}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)\left(e^{i\varphi}d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})-e^{-i\varphi}\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})\right)+$$

$$+2N_{ex}\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)(e^{2i\varphi}d^\dagger(\vec{K}-\vec{Q})d^\dagger(\vec{K}+\vec{Q})+e^{-2i\varphi}d(\vec{K}+\vec{Q})d(\vec{K}-\vec{Q})+$$

$$+2d^\dagger(\vec{K}-\vec{Q})d(\vec{K}-\vec{Q})-\frac{1}{N}\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}))- \quad (39)$$

$$-i\frac{N_{ex}}{N}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)Cos\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)(\hat{D}(\vec{Q})\hat{\rho}(-\vec{Q})-\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q}))+...$$

Below we will construct the motion equations for the operators (18) on the base of the Hamiltonian (37) in the quasiaverages theory approximation (QTA).

3. The motion equations for operators. Magneto-exciton-plasmon complexes.

The starting Hamiltonian $\hat{\mathcal{H}}$ of the variant developed below has the form

$$\hat{\mathcal{H}}=\frac{1}{2}\sum_{\vec{Q}}W_{\vec{Q}}\left[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})-\hat{N}_e-\hat{N}_h\right]-\mu_e\hat{N}_e-\mu_h\hat{N}_h+$$
$$+\sqrt{N_{ex}}(\bar{\mu}-E(\vec{k}))(e^{i\varphi}d^\dagger(\vec{k})+e^{-i\varphi}d(\vec{k}))-N_{ex}(\bar{\mu}-E(\vec{k}))\left(1-\frac{\hat{D}(0)}{N}\right) \quad (40)$$

The motion equations for the operators (18) are obtained using the commutation relations (20). They are

$$i\hbar\frac{d}{dt}d(P)=\left[d(P),\hat{\mathcal{H}}\right]=(E(p)-\tilde{\mu})d(\vec{P})-$$
$$-v\sqrt{N}e^{i\varphi}\delta_{kr}(\vec{P},\vec{K})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})+$$
$$+ve^{i\varphi}\left[iSin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{\rho}(\vec{P}-\vec{K})}{\sqrt{N}}+Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{D}(\vec{P}-\vec{K})}{\sqrt{N}}\right];$$

$$i\hbar\frac{d}{dt}d^\dagger(\vec{P})=[d^\dagger(\vec{P}),\hat{\mathcal{H}}]=(\tilde{\mu}-E(\vec{P}))d^\dagger(\vec{P})+v\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec{P},\vec{K})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\times$$
$$\times d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})+$$
$$+ve^{-i\varphi}\left[iSin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}-Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}\right];$$

$$i\hbar\frac{d}{dt}\hat{\rho}(\vec{P})=[\hat{\rho}(\vec{P}),\hat{\mathcal{H}}]=E(\vec{P})\hat{\rho}(\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\times$$
$$\times\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})+2iv\sqrt{N}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\left[e^{i\varphi}d^\dagger(\vec{K}-\vec{P})-e^{-i\varphi}d(\vec{P}+\vec{K})\right]; \quad (41)$$

$$i\hbar\frac{d}{dt}\hat{D}(\vec{P})=[\hat{D}(\vec{P}),\hat{\mathcal{H}}]=E(\vec{P})\hat{D}(\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\times$$

$$\times\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{Q})-2\nu\sqrt{N}Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\left[e^{i\varphi}d^{\dagger}(\vec{K}-\vec{P})-e^{-i\varphi}d(\vec{K}+\vec{P})\right].$$

Here ν and $\tilde{\mu}$ are determined by the expressions

$$\tilde{\mu}=\bar{\mu}(1-\frac{2N_{ex}}{N})+\frac{2N_{ex}}{N}E(\vec{k});$$

$$(\tilde{\mu}-E(\vec{k}))=(\bar{\mu}-E(\vec{k}))(1-2\mathrm{v}^2);\qquad \frac{N_{ex}}{N}=\mathrm{v}^2; \tag{42}$$

$$\nu=\sqrt{\frac{N_{ex}}{N}}(E(\vec{k})-\bar{\mu})=(E(\vec{k})-\bar{\mu})\mathrm{v};$$

The expression for ν was deduced in the previous section. The last term in the Hamiltonian $\hat{\mathcal{H}}$ (37) gives rise to the shift of the all exiton levels in the motion equations by the same value, what leads to the difference between $\bar{\mu}$ and $\tilde{\mu}$.

Now we must pay attention to one important aspect of the derived motion equations, which is closely related with the noncommutativity of the operators (18) expressed by formulas (20). Applying them one can prove, for example, the equalities

$$(E(\vec{P})-\tilde{\mu})d(\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})=$$

$$=-(E(\vec{P})+\tilde{\mu})d(\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)d(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})=$$

$$=-\tilde{\mu}\,d(P)-i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)(\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})+d(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})).$$

$$E(\vec{P})\hat{\rho}(\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})=$$

$$=-E(\vec{P})\hat{\rho}(\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})= \tag{43}$$

$$=-i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\left[\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})+\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})\right]$$

They can be verified taking into account that

$$2\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right) = E(P) \tag{44}$$

The quantum energy $E(P)$ is related with the vorticity of the strong magnetic field. These quanta can be added in different combinations to the free energies of the exciton or of the plasmon, which in their turn themselves also are determined by these quanta.

The quantum energy $E(P)$ is due to Coulomb interaction of electrons in the presence of a strong magnetic field and can be named as a plasmon quantum.

The possibility to add or to subtract a photon quantum from the electron energy in the presence of laser radiation gives rise to the notion of quasi – energy [7], which reflects the possibility of formation of electron – photon replicas.

In the same way one can understand the appearance of the different "free" magneto-exciton energies in three variants of the motion equations (43) as a result of formation of three different magneto-exciton-plasmon complexes: one with the "free" energy $(E(P)-\widetilde{\mu})$, the second with the "free" energy - $(E(P)+\widetilde{\mu})$ and the third with the "free" energy - $\widetilde{\mu}$. Starting with different "free" energies we will deal with the BEC of different magneto-exciton-plasmon complexes. In these three variants the constant ν of the broken gauge symmetry as well as $\widetilde{\mu}$ will be also different being conjugated with different "free" energies of the condensed particles. One can conclude that in the case of 2D e-h system in a strong perpendicular magnetic field the plasmon quanta (44) can accompany and influence the exciton quantum – statistical phenomena. In the case of the fractional quantum Hall effect (FQHE) discussed in [6], there are N magnetic flux quanta $\phi_0 = \frac{\hbar c}{e}$ accompanying the transport phenomena. The flux quanta enforce the formation of vortices in the 2D electron gas. The electrons and the vortices form composite particles and determine the properties of the electron liquid [6].

Instead of photons in the case of laser radiation and instead of magnetic flux quanta ϕ_0 and vortices in the electron medium, which appear in the case of FQHE, in our case of 2D–e-h system in a strong magnetic field we deal with plasmon quanta $E(P)$. Instead of electron – vortex composite particles we meet with the magneto-exciton-plasmon complexes. Equations (41) for four concrete interconnected operators $d^+(P), d(2\vec{K}-\vec{P}), \hat{\rho}(\vec{K}-\vec{P})$ and $\hat{D}(\vec{K}-\vec{P})$ have the forms

$$i\hbar\frac{d}{dt}d^\dagger(\vec{P}) = -(E(\vec{P})-\widetilde{\mu})d^\dagger(\vec{P}) + \nu\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec{P},\vec{K}) - 2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\times$$
$$\times d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q}) +$$
$$+\nu e^{-i\varphi}\left[iSin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}} - Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}\right];$$

$$
\begin{aligned}
& i\hbar\frac{d}{dt}d(2\vec{K}-\vec{P})=(E(2\vec{K}-\vec{P})-\widetilde{\mu})d(2\vec{K}-\vec{P})-v\sqrt{N}e^{i\varphi}\delta_{kr}(\vec{P},\vec{K})-\\
& -2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\times\hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})+\\
& +ve^{i\varphi}\left[-iSin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}+Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}\right];\\
& i\hbar\frac{d}{dt}\hat{\rho}(\vec{K}-\vec{P})=E(\vec{K}-\vec{P})\hat{\rho}(\vec{K}-\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\times\\
& \times\hat{\rho}(\vec{Q})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})-2iv\sqrt{N}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\left[e^{i\varphi}d^{\dagger}(\vec{P})-e^{-i\varphi}d(2\vec{K}-\vec{P})\right];\\
& i\hbar\frac{d}{dt}\hat{D}(\vec{K}-\vec{P})=E(\vec{K}-\vec{P})\hat{D}(\vec{K}-\vec{P})-2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\times\\
& \times\hat{\rho}(\vec{Q})\hat{D}(\vec{K}-\vec{P}-\vec{Q})-2v\sqrt{N}Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\left[e^{i\varphi}d^{\dagger}(\vec{P})-e^{-i\varphi}d(2\vec{K}-\vec{P})\right].
\end{aligned}
\tag{45}
$$

On the base of these equations the Green's functions will be introduced and the chains of equations will be developed. Only one variant between many ones reflected by equations (43) will be considered.

4. Many – operator many – particle Green's functions.

Following the motion equations (45) we will introduce four interconnected retarded Green's functions at T=0 [26, 27]

$$
\begin{aligned}
& \langle\langle d^{\dagger}(\vec{P},t);d(\vec{P},0)\rangle\rangle; && \langle\langle d(2\vec{K}-\vec{P},t);d(\vec{P},0)\rangle\rangle;\\
& \langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P},t)}{\sqrt{N}};d(\vec{P},0)\rangle\rangle; && \langle\langle\frac{\hat{D}(\vec{K}-\vec{P},t)}{\sqrt{N}};d(\vec{P},0)\rangle\rangle,
\end{aligned}
\tag{46}
$$

and their Fourier – transforms

$$
\begin{aligned}
& G_{11}(\vec{P},\omega)=\langle\langle d^{\dagger}(\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega}; && G_{12}(\vec{P},\omega)=\langle\langle d(2\vec{K}-\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega};\\
& G_{13}(\vec{P},\omega)=\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P}))}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}; && G_{14}(\vec{P},\omega)=\langle\langle\frac{\hat{D}(\vec{K}-\vec{P}))}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega};
\end{aligned}
\tag{47}
$$

They are determined by the relations

$$
\langle\langle\hat{A}(t);\hat{B}(0)\rangle\rangle=-i\Theta(t)\langle\left[\hat{A}(t),\hat{B}(0)\right]\rangle;\ A(t)=e^{\frac{i\hat{\mathcal{H}}t}{\hbar}}Ae^{-\frac{i\hat{\mathcal{H}}t}{\hbar}};\quad\left[\hat{A},\hat{B}\right]=\hat{A}\hat{B}-\hat{B}\hat{A},\tag{48}
$$

where $\hat{\mathcal{H}}$ is the Hamiltonian (40).

The average $\langle\rangle$ is calculated at T=0 using the ground state wave function $\left|\psi_g(\vec{K})\right\rangle$ (28) as well as, if needed, the coherent excited states (46)-(56) [4,5].

The time derivative of the Green's function is calculated as follows

$$i\hbar\frac{d}{dt}\langle\langle\hat{A}(t);\hat{B}(0)\rangle\rangle=\hbar\delta(t)\langle\left[\hat{A},\hat{B}\right]\rangle+\langle\langle i\hbar\frac{d}{dt}\hat{A}(t);\hat{B}(0)\rangle\rangle;$$
$$i\hbar\frac{d\hat{A}(t)}{dt}=\left[\hat{A}(t),\hat{\mathcal{H}}\right]=e^{\frac{i\hat{\mathcal{H}}t}{\hbar}}\left[\hat{A},\hat{\mathcal{H}}\right]e^{-\frac{i\hat{\mathcal{H}}t}{\hbar}} \tag{49}$$

The term $\langle\left[\hat{A},\hat{B}\right]\rangle$ and other similar ones will be denoted by constant C. The Fourier representation is introduced by

$$\langle\langle\hat{A}(t);\hat{B}(0)\rangle\rangle=\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\langle\langle\hat{A}\middle|\hat{B}\rangle\rangle_{\omega}$$
$$\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega t}d\omega \tag{50}$$
$$i\hbar\frac{d}{dt}\langle\langle\hat{A}(t);\hat{B}(0)\rangle\rangle=\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\hbar\omega\langle\langle\hat{A}\middle|\hat{B}\rangle\rangle_{\omega}$$

Calculating the Fourier transform of the retarded Green's function one needs to guarantee the convergence of the time integral. It is achieved by introducing an infinitesimal value $\delta\to+0$ in the form

$$\langle\langle\hat{A}\middle|\hat{B}\rangle\rangle_{\omega}=\int_{-\infty}^{\infty}dte^{i\omega t}\langle\langle\hat{A}(t);\hat{B}(0)\rangle\rangle=\int_{0}^{\infty}dte^{i\omega t-\delta t}\langle\langle\hat{A}(t);\hat{B}(0)\rangle\rangle \tag{51}$$

By this reason in the resonance denominators containing ω we will substitute $\hbar\omega$ by $(\hbar\omega+i\delta)$.

The Green's function (47) as well as (46) will be named as one – operator Green's functions because they contain in the left hand side of the vertical line only one summary operator of the type $d^{\dagger}(\vec{P}),d(2\vec{K}-\vec{P}),\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}$ and $\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}$. At the same time these Green's functions are two – particle Green's functions because the summary operators (18) are expressed through the products of two Fermi operators.

In this sense the Green's functions (47) are equivalent to the two – particle Green's functions introduced by Keldysh-Kozlov in their theory of the collective elementary excitations in bulk crystals in the absence of the external magnetic field [24].

The exact equations determining the one operator, two – particle Green's functions (47) in the frame of the quasi-average variant of the theory of BEC of magnetoexcitons follow directly from the motion equations (45)

$$\langle\langle d^{\dagger}(\vec{P})\middle|d(\vec{P})\rangle\rangle_{\omega}\left[\hbar\omega+E(P)-\tilde{\mu}\right]+$$
$$+2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_{z}l^{2}}{2}\right)\langle\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\middle|d(\vec{P})\rangle\rangle_{\omega}-$$
$$-ive^{-i\varphi}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_{z}l^{2}}{2}\right)\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\middle|d(\vec{P})\rangle\rangle_{\omega}+ve^{-i\varphi}Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_{z}l^{2}}{2}\right)\langle\langle\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}\middle|d(\vec{P})\rangle\rangle_{\omega}=C_{11};$$

$$\langle\langle d(2\vec{K}-\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega}\left[\hbar\omega - E(2\vec{K}-\vec{P})+\tilde{\mu}\right]+$$
$$+2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\langle\langle\hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\big|d(\vec{P})\rangle\rangle_{\omega}+$$
$$+ive^{i\varphi}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega} - ve^{i\varphi}Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\langle\langle\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega} = C_{12}$$

$$\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}\left[\hbar\omega - E(\vec{K}-\vec{P})\right]+$$
$$+2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\langle\langle\hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}+ \qquad (52)$$
$$+2ive^{i\varphi}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\langle\langle d^{\dagger}(\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega} - 2ive^{-i\varphi}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\langle\langle d(2\vec{K}-\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega} = C_{13}$$
$$\langle\langle\frac{\hat{D}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}\left[\hbar\omega - E(\vec{K}-\vec{P})\right]+$$
$$+2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\langle\langle\hat{\rho}(\vec{Q})\frac{\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}+$$
$$+2ve^{i\varphi}Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\langle\langle d^{\dagger}(\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega} - 2ve^{-i\varphi}Cos\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\langle\langle d(2\vec{K}-\vec{P})\big|d(\vec{P})\rangle\rangle_{\omega} = C_{14}$$

The constants C_{1i}, where $i=1,2,3,4$ depend on $\vec{P}$ and ω. But they are not needed in an explicit form, because we are interested only in the energy spectrum of the collective elementary excitations and it is determined only by the self – energy parts of the Green's functions. All constants, which will appear in the equations for any Green's functions will be denoted by C capital, without detalization.

Equations (52) for one-operator Green`s functions (47) contain in their componence the two-operator (four-particle) Green`s functions generated by the nonlinear terms in motion equations (41), (45) for the operators (18). These two-operator (four-particle) Green`s function will be determined below. They will obey to new exact equations in the frame of Hamiltonian (40) containing new three-operator (six-particle) Green`s functions. And this process is infinite giving rise to infinite chains of equations with n-operator (2n-particle) Green`s functions, where n increases by one at each new step in the chains evolution. As usual such chains are truncated, what leads to concrete approximate solutions [27]. Below we will obtain the exact equations in the frame of Hamiltonian (40) for four two-operator Green`s functions appeared in the first-step equations (35). These second step equations will contain new three-operators (six-particle) Grenn`s functions. They are

$$\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})\right]=C+\nu Ne^{-i\varphi}\delta_{kr}(\vec{Q},\vec{P}-\vec{K})\left\langle\left\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[(\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q}-\vec{R})\hat{\rho}(-\vec{R})\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+$$
$$+2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(-\vec{Q}-\vec{R})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-2i\nu\sqrt{N}e^{i\varphi}Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q})d^{\dagger}(\vec{K}+\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+$$
$$+2i\nu\sqrt{N}e^{-i\varphi}Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q})d(\vec{K}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+$$
$$+i\nu e^{-i\varphi}Sin\left(\frac{[(\vec{P}-\vec{Q})\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle\frac{\hat{\rho}(\vec{K}-\vec{P}+\vec{Q})}{\sqrt{N}}\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-\nu e^{-i\varphi}Cos\left(\frac{[(\vec{P}-\vec{Q})\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle\frac{\hat{D}(\vec{K}-\vec{P}+\vec{Q})}{\sqrt{N}}\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}; \quad (53)$$

$$\left\langle\left\langle\hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-\vec{E}(\vec{Q})\right]=C-\nu Ne^{i\varphi}\delta_{kr}(\vec{Q},\vec{K}-\vec{P})\left\langle\left\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[(2\vec{K}-\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})d(2\vec{K}-\vec{P}-\vec{Q}-\vec{R})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})d(2\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+$$
$$+2i\nu e^{i\varphi}\sqrt{N}Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(\vec{K}-\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-2i\nu e^{-i\varphi}\sqrt{N}Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle d(\vec{K}+\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-i\nu e^{i\varphi}Sin\left(\frac{[(\vec{P}+\vec{Q})\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle\frac{\hat{\rho}(\vec{Q})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+$$
$$+\nu e^{i\varphi}Cos\left(\frac{[(\vec{P}+\vec{Q})\times\vec{K}]_z l^2}{2}\right)\left\langle\left\langle\frac{\hat{\rho}(\vec{Q})\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}; \quad (54)$$

$$\left\langle\left\langle\frac{\hat{\rho}(\vec{Q})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\left[\hbar\omega-E(\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})\right]=C-$$
$$-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[(\vec{K}-\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle\frac{\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q}-\vec{R})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-$$
$$-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle\frac{\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+$$

$$
\begin{aligned}
&+2i\nu Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\Big[e^{i\varphi}\left\langle\left\langle d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-\\
&-e^{-i\varphi}\left\langle\left\langle d(\vec{Q}+\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\Big]-\\
&-2i\nu Sin\left(\frac{[(\vec{P}+\vec{Q})\times\vec{K}]_z l^2}{2}\right)\Big[e^{i\varphi}\left\langle\left\langle \hat{\rho}(\vec{Q})d^{\dagger}(\vec{P}+\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-\\
&-e^{-i\varphi}\left\langle\left\langle \hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\Big];
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
&\left\langle\left\langle \frac{\hat{\rho}(\vec{Q})\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\left[\hbar\omega-E(\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})\right]=C-\\
&-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[(\vec{K}-\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle \frac{\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\hat{D}(\vec{K}-\vec{P}-\vec{Q}-\vec{R})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-\\
&-2i\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\left\langle\left\langle \frac{\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+\\
&+2i\nu Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\Big[e^{i\varphi}\left\langle\left\langle d^{\dagger}(\vec{K}-\vec{Q})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-\\
&-e^{-i\varphi}\left\langle\left\langle d(\vec{K}+\vec{Q})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\Big]-\\
&-2\nu Cos\left(\frac{[(\vec{P}+\vec{Q})\times\vec{K}]_z l^2}{2}\right)\Big[e^{i\varphi}\left\langle\left\langle \hat{\rho}(\vec{Q})d^{\dagger}(\vec{P}+\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}-\\
&-e^{-i\varphi}\left\langle\left\langle \hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\Big].
\end{aligned}
\tag{56}
$$

As one can see, the second step equations (53)-(56) for the two-operator (four-particle) Green`s functions are exact what is the advantage of this method. They contain side by side with the three-operator Green`s functions other two-operator Greeen`s functions, for which in their turn the new equations must be deduced. It is one usual situation in the case of Green`s function method [13]. If one substitutes, for example, expression (53) for the two-operator Green`s function $\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}$ into the first equation (52) its contribution will be equal to

$$
\begin{aligned}
&2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}=\\
&C-\frac{2i\nu e^{-i\varphi}NW_{\vec{P}-\vec{K}}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{[\hbar\omega-\tilde{\mu}+E(\vec{K})-E(\vec{K}-\vec{P})+i\delta]}\left\langle\left\langle \frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}+
\end{aligned}
$$

$$+4\sum_{\vec{Q}}\sum_{\vec{R}}W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[(\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\frac{\left\langle\left\langle d^\dagger(\vec{P}-\vec{Q}-\vec{R})\hat{\rho}(-\vec{R})\hat{\rho}(-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_\omega}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}-$$

$$-4\sum_{\vec{Q}}\sum_{\vec{R}}W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\frac{\left\langle\left\langle d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(-\vec{Q}-\vec{R})\,|\,d(\vec{P})\right\rangle\right\rangle_\omega}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}+$$

$$+4ve^{i\varphi}\sqrt{N}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\frac{\left\langle\left\langle d^\dagger(\vec{P}-\vec{Q})d^\dagger(\vec{K}+\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_\omega}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}-$$

$$-4ve^{-i\varphi}\sqrt{N}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[\vec{Q}\times\vec{K}]_z l^2}{2}\right)\frac{\left\langle\left\langle d^\dagger(\vec{P}-\vec{Q})d(\vec{K}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_\omega}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}-$$

$$-2ve^{-i\varphi}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[(\vec{P}-\vec{Q})\times\vec{K}]_z l^2}{2}\right)\frac{\left\langle\left\langle \frac{\hat{\rho}(\vec{K}-\vec{P}+\vec{Q})\hat{\rho}(-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_\omega}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}-$$

$$-2ive^{-i\varphi}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Cos\left(\frac{[(\vec{P}-\vec{Q})\times\vec{K}]_z l^2}{2}\right)\frac{\left\langle\left\langle \frac{\hat{D}(\vec{K}-\vec{P}+\vec{Q})\hat{\rho}(-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_\omega}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}. \quad (57)$$

The-two operator Green's function (54) gives rise to the contribution to the second equation (52) in the form

$$2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\langle\langle\hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\Big|d(\vec{P})\rangle\rangle_\omega = C-$$

$$-\frac{2ive^{i\varphi}NW_{\vec{K}-\vec{P}}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\Big|d(\vec{P})\rangle\rangle_\omega+$$

$$+4\sum_{\vec{Q}}\sum_{\vec{R}}W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{Q}\times\vec{R}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})d(2\vec{K}-\vec{P}-\vec{Q})\Big|d(\vec{P})\rangle\rangle_\omega}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}+$$

$$+4\sum_{\vec{Q}}\sum_{\vec{R}}W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[(2\vec{K}-\vec{P}-\vec{Q})\times\vec{R}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})d(2\vec{K}-\vec{P}-\vec{Q}-\vec{R})\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}-$$

$$-4\nu\sqrt{N}e^{i\varphi}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{Q}\times\vec{K}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle d^{\dagger}(\vec{K}-\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}+$$

$$+4\nu\sqrt{N}e^{-i\varphi}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{Q}\times\vec{K}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle d^{\dagger}(\vec{Q}+\vec{K})d(2\vec{K}-\vec{P}-\vec{Q})\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}+$$

$$+2\nu e^{i\varphi}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[(\vec{P}+\vec{Q})\times\vec{K}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle\hat{\rho}(\vec{Q})\dfrac{\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}+$$

$$+2i\nu e^{i\varphi}\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Cos\left(\frac{\left[(\vec{P}+\vec{Q})\times\vec{K}\right]_z l^2}{2}\right)\times \tag{58}$$

$$\times\frac{\langle\langle\hat{\rho}(\vec{Q})\dfrac{\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}.$$

The contribution of the Green`s functions (55) and (56) to the third and fourth equations (52) are correspondingly

$$2i\sum_{\vec Q} W_{\vec Q} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)\left\langle\left\langle \hat\rho(\vec Q)\frac{\hat\rho(\vec K-\vec P-\vec Q)}{\sqrt N}\,|\,d(\vec P)\right\rangle\right\rangle_\omega =$$

$$C+4\sum_{\vec Q}\sum_{\vec R} W_{\vec Q}W_{\vec R} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[(\vec K-\vec P-\vec Q)\times\vec R]_z l^2}{2}\right)\frac{\left\langle\left\langle \hat\rho(\vec Q)\hat\rho(\vec R)\frac{\hat\rho(\vec K-\vec P-\vec Q-\vec R)}{\sqrt N}\,|\,d(\vec P)\right\rangle\right\rangle_\omega}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}+$$

$$+4\sum_{\vec Q}\sum_{\vec R} W_{\vec Q}W_{\vec R} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[\vec Q\times\vec R]_z l^2}{2}\right)\frac{\left\langle\left\langle \hat\rho(\vec R)\hat\rho(\vec Q-\vec R)\frac{\hat\rho(\vec K-\vec P-\vec Q)}{\sqrt N}\,|\,d(\vec P)\right\rangle\right\rangle_\omega}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}-$$

$$-4\nu\sum_{\vec Q} W_{\vec Q} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[\vec Q\times\vec K]_z l^2}{2}\right)\times$$

$$\times\frac{[e^{i\varphi}\left\langle\left\langle d^\dagger(\vec K-\vec Q)\hat\rho(\vec K-\vec P-\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega - e^{-i\varphi}\left\langle\left\langle d(\vec Q+\vec K)\hat\rho(\vec K-\vec P-\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega]}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}+$$

$$+4\nu\sum_{\vec Q} W_{\vec Q} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[(\vec P+\vec Q)\times\vec K]_z l^2}{2}\right)\times$$

$$\times\frac{[e^{i\varphi}\left\langle\left\langle \hat\rho(\vec Q)d^\dagger(\vec P+\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega - e^{-i\varphi}\left\langle\left\langle \hat\rho(\vec Q)d(2\vec K-\vec P-\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega]}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}; \qquad (59)$$

$$2i\sum_{\vec Q} W_{\vec Q} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)\left\langle\left\langle \hat\rho(\vec Q)\frac{\hat D(\vec K-\vec P-\vec Q)}{\sqrt N}\,|\,d(\vec P)\right\rangle\right\rangle_\omega =$$

$$C+4\sum_{\vec Q}\sum_{\vec R} W_{\vec Q}W_{\vec R} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[\vec Q\times\vec R]_z l^2}{2}\right)\frac{\left\langle\left\langle \frac{\hat\rho(\vec R)\hat\rho(\vec Q-\vec R)\hat D(\vec K-\vec P-\vec Q)}{\sqrt N}\,|\,d(\vec P)\right\rangle\right\rangle_\omega}{[\hbar\omega-E(\vec K-\vec P-\vec Q)-E(\vec Q)+i\delta]}+$$

$$+4\sum_{\vec Q}\sum_{\vec R} W_{\vec Q}W_{\vec R} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[(\vec K-\vec P-\vec Q)\times\vec R]_z l^2}{2}\right)\frac{\left\langle\left\langle \frac{\hat\rho(\vec Q)\hat\rho(\vec R)\hat D(\vec K-\vec P-\vec Q-\vec R)}{\sqrt N}\,|\,d(\vec P)\right\rangle\right\rangle_\omega}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}-$$

$$-4\nu\sum_{\vec Q} W_{\vec Q} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Sin\left(\frac{[\vec Q\times\vec K]_z l^2}{2}\right)\times$$

$$\times\frac{[e^{i\varphi}\left\langle\left\langle d^\dagger(\vec K-\vec Q)\hat D(\vec K-\vec P-\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega - e^{-i\varphi}\left\langle\left\langle d(\vec Q+\vec K)\hat D(\vec K-\vec P-\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega]}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}-$$

$$-4i\nu\sum_{\vec Q} W_{\vec Q} Sin\left(\frac{[(\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)Cos\left(\frac{[(\vec P+\vec Q)\times\vec K]_z l^2}{2}\right)\times$$

$$\times\frac{[e^{i\varphi}\left\langle\left\langle \hat\rho(\vec Q)d^\dagger(\vec P+\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega - e^{-i\varphi}\left\langle\left\langle \hat\rho(\vec Q)d(2\vec K-\vec P-\vec Q)\,|\,d(\vec P)\right\rangle\right\rangle_\omega]}{[\hbar\omega-E(\vec Q)-E(\vec K-\vec P-\vec Q)+i\delta]}. \qquad (60)$$

5. Decoupling of the elementary excitations. Shrinkage of the six-particle Green`s functions.

Expressions (57),(58),(59),(60) are too cumbersome to be prolonged in the same way because the three-operator Green`s functions will be expressed through the four-operator Green`s functions and so on. The shrinkage of the chains of Green`s functions can be achieved, if one will express the three operators, six-particle Green`s functions through the one-operator, two-particle Green`s functions as will be demonstrated below following the method of factorization elaborated in [27]. Another important simplification is the separation or the decoupling of the elementary excitations with different wave vectors as was proposed in [27, 28]. In the equations for the Green`s functions (30) only the terms containing the same Green`s functions are kept. The one-operator Green`s functions with other wave vectors different from $\vec{P},(2\vec{K}-\vec{P})$ and $(\vec{K}-\vec{P})$ are neglected. The two-operator Green`s functions will be expressed through the three-operator Green`s functions and the last will be approximatively expressed through the one-operator Green`s function multiplied by the average values of the remaining two operators. These approximations lead to the expressions:

$$\begin{aligned}
&\langle\langle d^\dagger(\vec{P}-\vec{Q}-\vec{R})\hat{\rho}(-\vec{R})\hat{\rho}(-\vec{Q})\big|d(\vec{P})\rangle\rangle_\omega \approx \delta_{kr}(\vec{R},-\vec{Q})\times\\
&\times\langle\langle d^\dagger(\vec{P})\big|d(\vec{P})\rangle\rangle_\omega\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle+\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_\omega\times\\
&\times\left[\delta_{kr}(\vec{R},\vec{P}-\vec{K})\left\langle d^\dagger(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle+\delta_{kr}(\vec{Q},\vec{P}-\vec{K})\left\langle d^\dagger(\vec{K}-\vec{R})\hat{\rho}(-\vec{R})\right\rangle\right]\sqrt{N};\\
&\langle\langle d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(-\vec{Q}-\vec{R})\big|d(\vec{P})\rangle\rangle_\omega \approx \delta_{kr}(\vec{Q},0)\langle\langle d^\dagger(\vec{P})\big|d(\vec{P})\rangle\rangle_\omega\times\\
&\times\left\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\right\rangle+\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_\omega\times\\
&\times\left[\delta_{kr}(\vec{R},\vec{K}-\vec{P})+\delta_{kr}(\vec{R},\vec{P}-\vec{K}-\vec{Q})\right]\left\langle d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})\right\rangle\sqrt{N};\\
&\langle\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})d(2\vec{K}-\vec{P}-\vec{Q})\big|d(\vec{P})\rangle\rangle_\omega \approx \delta_{kr}(\vec{Q},0)\langle\langle d^\dagger(2\vec{K}-\vec{P})\big|d(\vec{P})\rangle\rangle_\omega\times\\
&\times\left\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\right\rangle+\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_\omega\times\\
&\times\left[\delta_{kr}(\vec{R},\vec{K}-\vec{P})+\delta_{kr}(\vec{R},\vec{P}+\vec{Q}-\vec{K})\right]\left\langle\hat{\rho}(\vec{P}+\vec{Q}-\vec{K})d(2\vec{K}-\vec{P}-\vec{Q})\right\rangle\sqrt{N};\\
&\langle\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})d(2\vec{K}-\vec{P}-\vec{Q}-\vec{R})\big|d(\vec{P})\rangle\rangle_\omega \approx \delta_{kr}(\vec{R},-\vec{Q})\langle\langle d^\dagger(2\vec{K}-\vec{P})\big|d(\vec{P})\rangle\rangle_\omega\times\\
&\times\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle+\langle\langle\frac{\hat{\rho}(\vec{K}-\vec{P})}{\sqrt{N}}\big|d(\vec{P})\rangle\rangle_\omega\left[\delta_{kr}(\vec{Q},\vec{K}-\vec{P})\left\langle\hat{\rho}(\vec{R})d(\vec{K}-\vec{R})\right\rangle+\right.\\
&\left.+\delta_{kr}(\vec{R},\vec{K}-\vec{P})\left\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})\right\rangle\right]\sqrt{N}.
\end{aligned}\tag{61}$$

The decoupled and shrinked three-operator Green`s functions (61) being substituted into expressions (57) and (58) correspondingly will generate the following contributions to the desirable closed equations

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}} W_{\vec{R}} Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[(\vec{P}-\vec{Q})\times\vec{R}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle d^{\dagger}(\vec{P}-\vec{Q}-\vec{R})\hat{\rho}(-\vec{R})\hat{\rho}(-\vec{Q})\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}\approx$$

$$\approx -4\sum_{\vec{Q}} W_{\vec{Q}}{}^{2} Sin^{2}\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\frac{\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{\left[\hbar\omega-\bar{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}G_{11}(\vec{P},\omega)+$$

$$+G_{13}(\vec{P},\omega)\left\{4\sum_{\vec{Q}} W_{\vec{Q}} W_{\vec{P}-\vec{K}} Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[(\vec{P}-\vec{Q})\times(\vec{P}-\vec{K})\right]_z l^2}{2}\right)\times\right.$$

$$\times\frac{\left\langle d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle\sqrt{N}}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}-$$

$$\left.-\frac{4W_{\vec{P}-\vec{K}} Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega-\tilde{\mu}+E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}\sum_{\vec{Q}} W_{\vec{R}} Sin\left(\frac{\left[\vec{K}\times\vec{R}\right]_z l^2}{2}\right)\left\langle d^{\dagger}(\vec{K}-\vec{R})\hat{\rho}(-\vec{R})\right\rangle\sqrt{N}\right\};\qquad(62)$$

$$-4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}} W_{\vec{R}} Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[\vec{Q}\times\vec{R}\right]_z l^2}{2}\right)\frac{\langle\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(-\vec{Q}-\vec{R})\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}\approx$$

$$\approx -4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{K}-\vec{P}}-W_{\vec{P}-\vec{K}-\vec{Q}}) Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\times\qquad(63)$$

$$\times Sin\left(\frac{\left[\vec{Q}\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)\frac{\left\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}-\vec{K})\right\rangle\sqrt{N}}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}G_{13}(\vec{P},\omega);$$

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}} W_{\vec{R}} Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[\vec{Q}\times\vec{R}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})d(2\vec{K}-\vec{P}-\vec{Q})\big|d(\vec{P})\rangle\rangle_{\omega}}{\left[\hbar\omega+\tilde{\mu}+E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}\approx G_{13}(\vec{P},\omega)\times$$

$$\times 4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{K}-\vec{P}} - W_{\vec{P}+\vec{Q}-\vec{K}}) Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[\vec{Q}\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)\times$$

$$\times\frac{\left\langle\hat{\rho}(\vec{P}+\vec{Q}-\vec{K})d(2\vec{K}-\vec{P}-\vec{Q})\right\rangle\sqrt{N}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}; \quad (64)$$

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}} W_{\vec{R}} Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[(2\vec{K}-\vec{P}-\vec{Q})\times\vec{R}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\langle\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})d(2\vec{K}-\vec{P}-\vec{Q}-\vec{R})\big|d(\vec{P})\rangle\rangle_\omega}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}\approx$$

$$\approx -4\sum_{\vec{Q}} W_{\vec{Q}}{}^2 Sin^2\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\frac{\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]} G_{12}(\vec{P},\omega)+$$

$$+G_{13}(\vec{P},\omega)\left\{\frac{4W_{\vec{K}-\vec{P}} Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}\sum_{\vec{R}} W_{\vec{R}} Sin\left(\frac{\left[\vec{K}\times\vec{R}\right]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{R})d(\vec{K}-\vec{R})\right\rangle\sqrt{N}+\right.$$

$$+4\sum_{\vec{Q}} W_{\vec{Q}} W_{\vec{K}-\vec{P}} Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[(\vec{K}-\vec{Q})\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)\times$$

$$\left.\times\frac{\left\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{P})\right\rangle\sqrt{N}}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}\right\}. \quad (65)$$

The contributions (62)-(65) are proportional to Coulomb interaction in power two of the type $W_{\vec{Q}}W_{\vec{R}}$. Formulas (57) and (58) contain side by side with the three-operator Green`s functions also the two-operator Green`s functions. The latter are incorporated into the terms proportional to $\nu W_{\vec{Q}}$. After their expression through the three-operator Green`s functions and the following transformation into the one-operator Green`s functions their contribution will be proportional to $\nu W_{\vec{Q}}{}^2$.

The constant ν and its dependence on μ in our case was determined above. It is of the same type as (33) and its dependence on the Coulomb interaction originates from the dependence of μ, which was determined in [4]. In the Hartree-Fock-Bogoliubov approximation the chemical potential has the value [4]

$$\overline{\mu}^{HF} - E(K) = 2\nu^2 E(K)$$

and depends linearly on the constant of the Coulomb interaction, whereas its correlation corrections are quadratic on this interaction. In all our next calculations we will confine ourselves to the self-energy parts linear and quadratic in Coulomb interaction. In these restrictions one can neglect all the terms containing two-operator Green`s functions in formulas (57)-(60) because their investements will be of the order $\nu W_{\vec{Q}}^{\,2}$. By the same reason we will neglect the terms proportional to $\nu^2 W_{\vec{Q}}$.

As a result in our present variant of the paper we will take into account the terms proportional to $\nu, W_{\vec{Q}}, \nu W_{\vec{Q}}$ and $W_{\vec{Q}}^{\,2}$ and will neglect the terms proportional to $W_{\vec{Q}}^{\,3}, \nu W_{\vec{Q}}^{\,2}$ and $\nu^2 W_{\vec{Q}}$. In this approximation the nonlinear terms of the first and second equations (52) are

$$2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\langle\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\big| d(\vec{P})\rangle\rangle_{\omega} \approx$$

$$\approx C - 4\sum_{\vec{Q}} W_{\vec{Q}}^{\,2} Sin^2\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)\frac{\left\langle \hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{\left[\hbar\omega - \widetilde{\mu} + E(\vec{P}-\vec{Q}) - E(-\vec{Q}) + i\delta\right]} G_{11}(\vec{P},\omega) +$$

$$+G_{13}(\vec{P},\omega)\left\{-\frac{2i\nu e^{-i\varphi}(W_{\vec{P}-\vec{K}}N) Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega - \widetilde{\mu} + E(\vec{K}) - E(\vec{K}-\vec{P}) + i\delta\right]} + \right.$$

$$+4\sum_{\vec{Q}} W_{\vec{Q}} W_{\vec{P}-\vec{K}} Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right) Sin\left(\frac{\left[(\vec{P}-\vec{Q})\times(\vec{P}-\vec{K})\right]_z l^2}{2}\right)\frac{\left\langle d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle\sqrt{N}}{\left[\hbar\omega - \widetilde{\mu} + E(\vec{P}-\vec{Q}) - E(-\vec{Q}) + i\delta\right]} -$$

$$-\frac{4W_{\vec{P}-\vec{K}} Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega - \widetilde{\mu} + E(\vec{K}) - E(\vec{K}-\vec{P}) + i\delta\right]}\sum_{\vec{R}} W_{\vec{R}} Sin\left(\frac{\left[\vec{K}\times\vec{R}\right]_z l^2}{2}\right)\left\langle d^{\dagger}(\vec{K}-\vec{R})\hat{\rho}(-\vec{R})\right\rangle\sqrt{N} -$$

$$-4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{K}-\vec{P}}-W_{\vec{P}-\vec{K}-\vec{Q}})Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{Q}\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)\times$$

$$\left.\times\frac{\left\langle d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}-\vec{K})\right\rangle\sqrt{N}}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}\right\}; \quad (66)$$

$$2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\langle\langle\hat{\rho}(\vec{Q})d(2\vec{K}-\vec{P}-\vec{Q})\big|d(\vec{P})\rangle\rangle_{\omega}\approx$$

$$\approx C-4\sum_{\vec{Q}} W_{\vec{Q}}{}^{2} Sin^2\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\frac{\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}G_{12}(\vec{P},\omega)+$$

$$+G_{13}(\vec{P},\omega)\left\{-\frac{2ive^{i\varphi}(W_{\vec{K}-\vec{P}}N)Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}+\right.$$

$$+4\sum_{\vec{Q}} W_{\vec{Q}}(W_{\vec{K}-\vec{P}}-W_{\vec{P}+\vec{Q}-\vec{K}})\frac{Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{Q}\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}\times$$

$$\times\left\langle\hat{\rho}(\vec{P}+\vec{Q}-\vec{K})d(2\vec{K}-\vec{P}-\vec{Q})\right\rangle\sqrt{N}+$$

$$+\frac{4W_{\vec{K}-\vec{P}}Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}\sum_{\vec{R}} W_{\vec{R}} Sin\left(\frac{\left[\vec{K}\times\vec{R}\right]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{R})d(\vec{K}-\vec{R})\right\rangle\sqrt{N}+ \quad (67)$$

$$\left.+4\sum_{\vec{Q}} W_{\vec{Q}}W_{\vec{K}-\vec{P}}\frac{Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[(\vec{K}-\vec{Q})\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)}{\left[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}\left\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})\right\rangle\sqrt{N}\right\}$$

Now the two first equations (52) can be written in the forms

$$G_{11}(\vec{P},\omega)\sum{}_{11}(P,\omega)+G_{12}(\vec{P},\omega)\sum{}_{21}(P,\omega)+G_{13}(\vec{P},\omega)\sum{}_{31}(P,\omega)+$$

$$+G_{14}(\vec{P},\omega)\sum{}_{41}(P,\omega)=C_{11}$$

$$G_{11}\Sigma_{12}(\vec{P},\omega)+G_{12}\Sigma_{22}(\vec{P},\omega)+G_{13}\Sigma_{32}(\vec{P},\omega)+G_{14}\Sigma_{42}(\vec{P},\omega)=C_{12} \tag{68}$$

Their self-energy parts are determined by the following expressions:

$$\Sigma_{11}(\vec{P},\omega)=\hbar\omega-\tilde{\mu}+E(\vec{P})-4\sum_{\vec{Q}}W_{\vec{Q}}^2\frac{Sin^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle;$$

$$\Sigma_{21}(\vec{P},\omega)=0;$$

$$\begin{aligned}\Sigma_{31}(\vec{P},\omega)&=-ive^{-i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\left[1+\frac{2W_{\vec{P}-\vec{K}}N}{[\hbar\omega-\tilde{\mu}+E(\vec{K})-E(\vec{K}-\vec{P})+i\delta]}\right]+\\&+4\sum_{\vec{Q}}W_{\vec{Q}}W_{\vec{P}-\vec{K}}\frac{Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[(\vec{P}-\vec{Q})\times(\vec{P}-\vec{K})]_z l^2}{2}\right)}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}\left\langle d^\dagger(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle\sqrt{N}-\\&-4W_{\vec{P}-\vec{K}}\frac{Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{[\hbar\omega-\tilde{\mu}+E(\vec{K})-E(\vec{K}-\vec{P})+i\delta]}\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{K}\times\vec{R}]_z l^2}{2}\right)\left\langle d^\dagger(\vec{K}-\vec{R})\hat{\rho}(-\vec{R})\right\rangle\sqrt{N}-\\&-4\sum_{\vec{Q}}W_{\vec{Q}}\frac{\left(W_{\vec{K}-\vec{P}}-W_{\vec{P}-\vec{K}-\vec{Q}}\right)Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[\vec{Q}\times(\vec{K}-\vec{P})]_z l^2}{2}\right)}{[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta]}\left\langle d^\dagger(\vec{P}-\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}-\vec{K})\right\rangle\sqrt{N};\end{aligned} \tag{69}$$

$$\Sigma_{41}(\vec{P},\omega)=ve^{-i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right).$$

The self-energy parts $\Sigma_{i1}(\vec{P},\omega)$ determine the coefficients of the first equation (68). The self-energy parts $\Sigma_{i2}(\vec{P},\omega)$ of the second equation (68) are

$$\Sigma_{12}(\vec{P},\omega)=0;$$

$$\Sigma_{22}(\vec{P},\omega)=\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P})-$$

$$-4\sum_{\vec{Q}}W_{\vec{Q}}^2\frac{Sin^2\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta]};$$

$$\Sigma_{32}(\vec{P},\omega)=i\nu e^{i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\left[1-\frac{2W_{\vec{K}-\vec{P}}N}{[\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta]}\right]+$$

$$+4\sum_{\vec{Q}}W_{\vec{Q}}(W_{\vec{K}-\vec{P}}-W_{\vec{P}+\vec{Q}-\vec{K}})Sin\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\frac{Sin\left(\frac{[\vec{Q}\times(\vec{K}-\vec{P})]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{P}+\vec{Q}-\vec{K})d(2\vec{K}-\vec{P}-\vec{Q})\right\rangle\sqrt{N}}{[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta]}+ \quad (70)$$

$$+4\sum_{\vec{Q}}W_{\vec{Q}}\frac{W_{\vec{K}-\vec{P}}Sin\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[(\vec{K}-\vec{Q})\times(\vec{K}-\vec{P})]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{Q})d(\vec{K}-\vec{Q})\right\rangle\sqrt{N}}{[\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta]}+$$

$$+4W_{\vec{K}-\vec{P}}\frac{Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)}{[\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta]}\sum_{\vec{R}}W_{\vec{R}}Sin\left(\frac{[\vec{K}\times\vec{R})]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{R})d(\vec{K}-\vec{R})\right\rangle\sqrt{N};$$

$$\Sigma_{42}(\vec{P},\omega)=-\nu e^{i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right).$$

Now the remaining two equations (52) will be considered. The reduction of three-operator Green`s functions encountered in the nonlinear terms (59) and (60) is made as follows

$$\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\frac{\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\simeq G_{13}(\vec{P},\omega)[(\delta_{kr}(\vec{R},\vec{K}-\vec{P})+$$
$$+\delta_{kr}(\vec{R},\vec{Q}+\vec{P}-\vec{K}))\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\right\rangle+\delta_{kr}(\vec{Q},0)\left\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\right\rangle] \;;$$

$$\left\langle\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q}-\vec{R})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\simeq G_{13}(\vec{P},\omega)[(\delta_{kr}(\vec{R},\vec{K}-\vec{P})+$$
$$+\delta_{kr}(\vec{R},-\vec{Q}))\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle+\delta_{kr}(\vec{Q},\vec{K}-\vec{P})\left\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\right\rangle] \;;$$

$$\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\frac{\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\simeq G_{13}(\vec{P},\omega)(\delta_{kr}(\vec{R},\vec{K}-\vec{P})+ \quad (71)$$
$$+\delta_{kr}(\vec{R},\vec{Q}+\vec{P}-\vec{K}))\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\right\rangle+G_{14}(\vec{P},\omega)\delta_{kr}(\vec{Q},0)\left\langle\hat{\rho}(\vec{R})\hat{\rho}(-\vec{R})\right\rangle$$

$$\left\langle\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\frac{\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\simeq G_{13}(\vec{P},\omega)[(\delta_{kr}(\vec{Q},\vec{K}-\vec{P})\left\langle\hat{\rho}(\vec{R})\hat{D}(-\vec{R})\right\rangle+$$
$$+\delta_{kr}(\vec{R},\vec{K}-\vec{P}))\left\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\right\rangle]+G_{14}(\vec{P},\omega)\delta_{kr}(\vec{R},-\vec{Q})\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle.$$

They lead to approximate expression of two main components of (42)

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\frac{\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]\sqrt{N}}\approx$$

$$\approx G_{13}(\vec{P},\omega)4\sum_{\vec{Q}}W_{\vec{Q}}\frac{(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}})Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}; \quad (72)$$

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[(\vec{K}-\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\frac{\left\langle\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q}-\vec{R})\,|\,d(\vec{P})\right\rangle\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]\sqrt{N}}\approx$$

$$\approx G_{13}(\vec{P},\omega)4\sum_{\vec{Q}}W_{\vec{Q}}(W_{\vec{K}-\vec{P}}-W_{-\vec{Q}})Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle.$$

Taking into account only the terms proportional to $W_{\vec{Q}}W_{\vec{R}}$ and neglecting the last two terms in (59) because they give the contributions to the one-operator Green`s functions $G_{i1}(\vec{P},\omega)$ proportional to $\nu W_{\vec{Q}}^2$ one will obtain

$$2i\sum_{\vec{Q}}W_{\vec{Q}}Sin\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\left\langle\left\langle\hat{\rho}(\vec{Q})\frac{\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}}\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}\approx$$

$$\approx C+G_{13}(\vec{P},\omega)4\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\times \quad (73)$$

$$\times\frac{[(W_{\vec{K}-\vec{P}}-W_{-\vec{Q}})\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle+(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}})\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\right\rangle]}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}.$$

The same processing will be made with the contribution (60). Here we have obtained

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[\vec{Q}\times\vec{R}]_z l^2}{2}\right)\frac{\left\langle\left\langle\hat{\rho}(\vec{R})\hat{\rho}(\vec{Q}-\vec{R})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}}{[\hbar\omega-E(\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta]\sqrt{N}}\approx$$

$$\approx G_{13}(\vec{P},\omega)4\sum_{\vec{Q}}W_{\vec{Q}}(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}})Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\times$$

$$\times\frac{\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\right\rangle}{[\hbar\omega-E(\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta]};$$

$$4\sum_{\vec{Q}}\sum_{\vec{R}} W_{\vec{Q}}W_{\vec{R}}Sin\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)Sin\left(\frac{[(\vec{K}-\vec{P}-\vec{Q})\times\vec{R}]_z l^2}{2}\right)\frac{\left\langle\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(\vec{R})\hat{D}(\vec{K}-\vec{P}-\vec{Q}-\vec{R})\,|\,d(\vec{P})\right\rangle\right\rangle_{\omega}}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]\sqrt{N}}\approx$$

$$\approx G_{13}(\vec{P},\omega)4\sum_{\vec{Q}}W_{\vec{Q}}^2Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\frac{\left\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})]\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}- \quad (74)$$

$$-G_{14}(\vec{P},\omega)4\sum_{\vec{Q}}W_{\vec{Q}}^2Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\frac{\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})]\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}.$$

In the same approximation as was applied to (59), the nonlinear term will be determined as equal to

$$2i\sum_{\vec{Q}} W_{\vec{Q}} Sin\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\left\langle\left\langle \hat{\rho}(\vec{Q})\frac{\hat{D}(\vec{K}-\vec{P}-\vec{Q})}{\sqrt{N}} \mid d(\vec{P})\right\rangle\right\rangle_{\omega} \approx$$

$$\approx C(\omega)+G_{13}(\vec{P},\omega)4\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\times$$
$$\times\frac{[(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}})\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\right\rangle+W_{\vec{Q}}\left\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\right\rangle]}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}- \tag{75}$$
$$G_{14}(\vec{P},\omega)4\sum_{\vec{Q}} W^2_{\vec{Q}} Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\frac{\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}.$$

Now we will substitute the nonlinear terms (74) and (75) into the third and fourth equations (52) correspondingly. These two equations can be written in the forms

$$G_{11}(\vec{P},\omega)\Sigma_{13}(\vec{P},\omega)+G_{12}(\vec{P},\omega)\Sigma_{23}(\vec{P},\omega)+G_{13}(\vec{P},\omega)\Sigma_{33}(\vec{P},\omega)+$$
$$+G_{14}(\vec{P},\omega)\Sigma_{43}(\vec{P},\omega)=C_{13} \tag{76}$$
$$G_{11}(\vec{P},\omega)\Sigma_{14}(\vec{P},\omega)+G_{12}(\vec{P},\omega)\Sigma_{24}(\vec{P},\omega)+G_{13}(\vec{P},\omega)\Sigma_{34}(\vec{P},\omega)+$$
$$+G_{14}(\vec{P},\omega)\Sigma_{44}(\vec{P},\omega)=C_{14}$$

Their self-energy parts are:

$$\Sigma_{13}(\vec{P},\omega)=2ive^{i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$
$$\Sigma_{23}(\vec{P},\omega)=-2ive^{-i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$
$$\Sigma_{33}(\vec{P},\omega)=\hbar\omega-E(\vec{K}-\vec{P})+4\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\times$$
$$\times\frac{[(W_{\vec{K}-\vec{P}}-W_{-\vec{Q}})\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle+(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}})\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q})\right\rangle]}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]};$$
$$\Sigma_{43}(0)=0;$$
$$\Sigma_{14}(\vec{P},\omega)=2ve^{i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right); \tag{77}$$
$$\Sigma_{24}(\vec{P},\omega)=-2ve^{-i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$

$$\Sigma_{34}(\vec{P},\omega)=4\sum_{\vec{Q}} W_{\vec{Q}} Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\times$$
$$\times\frac{[(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}})\left\langle\hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{D}(\vec{K}-\vec{P}-\vec{Q})\right\rangle+W_{\vec{Q}}\left\langle\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q})\right\rangle]}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]};$$
$$\Sigma_{44}(\vec{P},\omega)=\hbar\omega-E(\vec{K}-\vec{P})-4\sum_{\vec{Q}} W_{\vec{Q}}^2 Sin^2\left(\frac{[(\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right)\times$$
$$\times\frac{\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle}{[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta]}.$$

The self-energy part (69) and (77) determine the full set of self-energy parts in the approximation which is equivalent to the taking into account of the correlation energy in the frame of coherent excited states discussed in [4,5] beyond the Hartree-Fock-Bogoliubov (HFB) approximation. But before we will study the energy spectrum in a simpler approach.

6. Energy spectrum in the Hartree-Fock-Bogoliubov approximation.

The exact equations for the Green`s functions (47) following expressions (52), (57)-(60) contains terms of type $E(p)$ linear in the Coulomb interaction $W_{\vec{Q}}$, the terms of the type $W_{\vec{Q}}W_{\vec{R}}$ quadratic in the Coulomb interaction as well as the mixed terms $\nu W_{\vec{Q}}$, where the constant ν characterizing the broken symmetry is proportional to $(E(k)-\bar{\mu})$, if the BEC of magnetoexcitons takes place on the state with $\vec{k}\neq 0$. The last relation was established in (42). The chemical potential μ was deduced in [4,5] and it also contains terms proportional to $W_{\vec{Q}}$ and $W_{\vec{Q}}{}^2$. In the Hartree-Fock-Bogoliubov (HFB) approximation we will confine ourselves only with the terms linear in Coulomb interaction $W_{\vec{Q}}$. The possibility of such approach must be verified posteriory. If so, equation (52) for the Green`s functions (47) will take the simple forms of Dyson equations with zeroth-order, or (HFB) self-energy parts $\sum_{ij}^{HF}(P,\omega)$ as follows

$$G_{11}^{(P,\omega)}\sum\nolimits_{1i}^{HF}(P,\omega)+G_{12}^{(P,\omega)}\sum\nolimits_{2i}^{HF}(P,\omega)+G_{13}^{(P,\omega)}\sum\nolimits_{3i}^{HF}(P,\omega)+$$
$$G_{14}^{(P,\omega)}\sum\nolimits_{4i}^{HF}(P,\omega)=C_{1i}; \tag{78}$$
$$i=1,2,3,4$$

If one will introduce the Green`s functions and self-energy parts in the matrix forms and if we will add a matrix formed by the coefficients C_{ij}

$$\hat{G}(\vec{P},\omega)=\left\|G_{ij}(\vec{P},\omega)\right\|;\qquad \hat{\Sigma}^{HF}(\vec{P},\omega)=\left\|\sum\nolimits_{ij}^{HF}(\vec{P},\omega)\right\|\qquad \hat{C}=\left\|C_{ij}\right\| \tag{79}$$

it will permit to write the Dyson equation in a matrix form

$$\hat{G}(\vec{P},\omega)\hat{\Sigma}^{HF}(\vec{P},\omega)=\hat{C} \tag{80}$$

Equations (68) coincide with one part of equations (80). The other part is not needed because they lead to the same dispersion equation as equations (52) do. The self-energy parts $\sum_{ij}^{HF}(\vec{P},\omega)$ introduced into formulas (54), (55), (56) are listed below

$$\sum_{11}^{HF}(\vec{P},\omega)=\hbar\omega+E(\vec{P})-\tilde{\mu}^{HF}; \qquad \sum_{21}^{HF}(\vec{P},\omega)=0;$$

$$\sum_{31}^{HF}(\vec{P},\omega)=-i\nu^{HF}e^{-i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right); \qquad \sum_{41}^{HF}(\vec{P},\omega)=\nu^{HF}e^{-i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$

$$\sum_{12}^{HF}(\vec{P},\omega)=0; \qquad \sum_{22}^{HF}(\vec{P},\omega)=\hbar\omega+\tilde{\mu}^{HF}-E(2\vec{K}-\vec{P}); \tag{81}$$

$$\sum_{32}^{HF}(\vec{P},\omega)=i\nu^{HF}e^{i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right); \qquad \sum_{42}^{HF}(\vec{P},\omega)=-\nu^{HF}e^{i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$

$$\sum_{13}^{HF}(\vec{P},\omega)=2i\nu^{HF}e^{i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right); \qquad \sum_{23}^{HF}(\vec{P},\omega)=-2i\nu^{HF}e^{-i\varphi}Sin\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$

$$\sum_{33}^{HF}(\vec{P},\omega)=\hbar\omega-E(\vec{K}-\vec{P}); \qquad \sum_{43}^{HF}(\vec{P},\omega)=0;$$

$$\sum_{14}^{HF}(\vec{P},\omega)=2\nu^{HF}e^{i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right); \qquad \sum_{24}^{HF}(\vec{P},\omega)=-2\nu^{HF}e^{-i\varphi}Cos\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right);$$

$$\sum_{44}^{HF}(\vec{P},\omega)=\hbar\omega-E(\vec{K}-\vec{P}); \qquad \sum_{34}^{HF}(\vec{P},\omega)=0.$$

The values $\tilde{\mu}$ and ν of the zeroth order are denoted by $\tilde{\mu}^{HF}$ and ν^{HF}. It corresponds to the Hartree-Fock-Bogoliubov (HFB) approximation. Following formula (42) [4] $\bar{\mu}^{HF}-E(k)=-2\nu^2E(k)$

The energy spectrum is determined by the solution of the determinant equation

$$\det\left\|\sum_{ij}^{HF}(P,\omega)\right\|=0 \tag{82}$$

which has the form

$$\begin{vmatrix} \sum_{11}^{HF}(P,\omega) & 0 & \sum_{31}^{HF}(P,\omega) & \sum_{41}^{HF}(P,\omega) \\ 0 & \sum_{22}^{HF}(P,\omega) & \sum_{32}^{HF}(P,\omega) & \sum_{42}^{HF}(P,\omega) \\ \sum_{13}^{HF}(P,\omega) & \sum_{23}^{HF}(P,\omega) & \sum_{33}^{HF}(P,\omega) & 0 \\ \sum_{14}^{HF}(P,\omega) & \sum_{24}^{HF}(P,\omega) & 0 & \sum_{44}^{HF}(P,\omega) \end{vmatrix}=0 \tag{83}$$

The expressions $\sum_{ij}^{HF}(\vec{P},\omega)$ obey one exact relation

$$\begin{aligned}&\sum_{31}^{HF}(P,\omega)\sum_{13}^{HF}(P,\omega)\sum_{42}^{HF}(P,\omega)\sum_{24}^{HF}(P,\omega)+\sum_{32}^{HF}(P,\omega)\sum_{23}^{HF}(P,\omega)\sum_{41}^{HF}(P,\omega)\sum_{14}^{HF}(P,\omega)-\\&-\sum_{31}^{HF}(P,\omega)\sum_{23}^{HF}(P,\omega)\sum_{42}^{HF}(P,\omega)\sum_{14}^{HF}(P,\omega)-\sum_{32}^{HF}(P,\omega)\sum_{13}^{HF}(P,\omega)\sum_{41}^{HF}(P,\omega)\sum_{24}^{HF}(P,\omega)=0\end{aligned} \tag{84}$$

It leads to simplification of the dispersion relation (83), which will take the form

$$\sum_{11}^{HF}(P,\omega)\sum_{22}^{HF}(P,\omega)\sum_{33}^{HF}(P,\omega)\sum_{44}^{HF}(P,\omega)-$$
$$-2(\nu^{HF})^2\left(\sum_{11}^{HF}(P,\omega)\sum_{33}^{HF}(P,\omega)+\sum_{22}^{HF}(P,\omega)\sum_{44}^{HF}(P,\omega)\right)=0 \quad (85)$$

Due to the equality $\sum_{33}^{HF}(P,\omega)=\sum_{44}^{HF}(P,\omega)$, this dispersion relation can be factorized and two relations can be written. One of them describes the simple plasmon solution

$$\sum_{33}^{HF}(P)=0; \quad \hbar\omega=E(\vec{K}-\vec{P}) \quad (86)$$

The other one is the third order equation

$$\sum_{11}^{HF}(P,\omega)\sum_{22}^{HF}(P,\omega)\sum_{44}^{HF}(P,\omega)-2(\nu^{HF})^2\left(\sum_{11}^{HF}(P,\omega)+\sum_{22}^{HF}(P,\omega)\right)=0 \quad (87)$$

which takes the form

$$\left[(\hbar\omega)^2-(\tilde{\mu}^{HF})^2-E(\vec{P})E(2\vec{K}-\vec{P})+\hbar\omega(E(\vec{P})-E(2\vec{K}-\vec{P}))+\tilde{\mu}^{HF}(E(\vec{P})+E(2\vec{K}-\vec{P}))\right]\times$$
$$\times(\hbar\omega-E(\vec{K}-\vec{P}))-2(\nu^{HF})^2(2\hbar\omega+E(\vec{P})-E(2\vec{K}-\vec{P}))=0 \quad (88)$$

In the special case $\vec{P}=\vec{K}$ it looks as

$$(\hbar\omega)^3-\hbar\omega\left[(E(\vec{K})-\tilde{\mu}^{HF})^2+4(\nu^{HF})^2\right]=0$$

and has three solutions

$$\hbar\omega_1(\vec{P}=\vec{K})=0$$
$$\hbar\omega_{2,3}(\vec{P}=\vec{K})=\pm\sqrt{(E(\vec{K})-\tilde{\mu}^{HF})^2+4(\nu^{Hf})^2} \quad (89)$$

Now the more general case will be considered introducing the small deviation of the vector $\vec{P}$ from the condensate wave vector $\vec{K}$ in the form $\vec{P}=\vec{K}+\vec{q}$ and using the series expansions on the small wave vector $\vec{q}$ as follows

$$E(\vec{P})=E(\vec{K}+\vec{q})=E(\vec{K})+\hbar\vec{\mathrm{v}}_{\mathrm{g}}(\vec{K})\vec{q}+\frac{\hbar^2q^2}{2M(\vec{K})}$$
$$\vec{\mathrm{v}}_{\mathrm{g}}(\vec{K})=\frac{\partial E(\vec{K})}{\partial\vec{K}}; \qquad M(\vec{K})=\frac{\hbar^2}{\frac{\partial^2E(\vec{K})}{(\partial K)^2}}; \quad (90)$$
$$E(\vec{K}-\vec{P})=E(\vec{q}); \qquad E(2\vec{K}-\vec{P})=E(\vec{K}-\vec{q})$$

Here the group velocity $\vec{\mathrm{v}}_{\mathrm{g}}(\vec{K})$ and the magnetic mass $M(\vec{K})$ at the condensate wave vector $\vec{K}$ are introduced.

Then the coefficients of equation (88) will become

$$E(\vec{P}) - E(2\vec{K} - \vec{P}) = 2\hbar \mathrm{v}_{\mathrm{g}}(\vec{K})\vec{q}$$

$$\left(E(\vec{P}) - \tilde{\mu}^{HF} \right)\left(E(2\vec{K} - \vec{P}) - \tilde{\mu}^{HF} \right) = \left(E(\vec{K}) - \tilde{\mu}^{HF} + \frac{\hbar^2 q^2}{2M(\vec{K})} \right)^2 - \left(\hbar \mathrm{v}_{\mathrm{g}}(\vec{K})\vec{q} \right)^2 ; \quad (91)$$

$$E(\vec{K} - \vec{P})\left(E(\vec{P}) - E(2\vec{K} - \vec{P}) \right) = 2E(\vec{q})\hbar \mathrm{v}_{\mathrm{g}}(\vec{K})\vec{q}$$

The third order dispersion equation (88) looks as complete cubic equation

$$(\hbar\omega)^3 + (\hbar\omega)^2\left[2\hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} - E(q)\right] - \hbar\omega\left[\left(E(\vec{K}) - \tilde{\mu}^{HF} + \frac{\hbar^2 q^2}{2M(\vec{K})} \right)^2 - \left(\hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q}\right)^2 + 2E(q)\hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} + 4\left(\nu^{HF}\right)^2\right] +$$
$$E(q)\left[\left(E(K) - \tilde{\mu}^{HF} + \frac{\hbar^2 q^2}{2M(\vec{K})} \right)^2 - \left(\hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q}\right)^2\right] - 4\left(\nu^{HF}\right)^2 \hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} = 0 \quad (92)$$

It can be transformed by the substitution

$$\hbar\omega(q) = y(q) + \frac{1}{3}\left(E(q) - 2\hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} \right) \quad (93)$$

into the incomplete cubic equation

$$y^3 + py + g = 0 \quad (94)$$

with the coefficients $p(q)$ and $g(q)$

$$p(q) = -\left[\left(E(\vec{K}) - \tilde{\mu}^{HF} + \frac{\hbar^2 q^2}{2M(\vec{K})} \right)^2 + 4\left(\nu^{HF}\right)^2 + \frac{1}{3}\left(\hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} + E(q)\right)^2\right] < 0 \quad (95)$$

$$g(q) = \frac{2}{3}\left(E(q) + \hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} \right)\left[\left(E(\vec{K}) - \tilde{\mu}^{HF} + \frac{\hbar^2 q^2}{2M(\vec{K})} \right)^2 - 2\left(\nu^{HF}\right)^2 - \frac{1}{9}\left(E(q) + \hbar\vec{\mathrm{v}}_{\mathrm{g}}\vec{q} \right)^2\right] \quad (96)$$

Because $p(q) < 0$, the value $Q = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2$ can be negative, if $\left(-\frac{p}{3}\right)^3 > \left(\frac{q}{2}\right)^2$. In this irreducible case there are trigonometric solutions for three real roots of equation (94). They are [29]

$$y_1(q) = 2\sqrt{-p(q)/3}\, Cos\frac{\alpha(q)}{3}$$
$$y_{2,3} = -2\sqrt{-p(q)/3}\, Cos\left(\frac{\alpha(q)}{3} \pm \frac{2\pi}{3} \right) \quad (97)$$
$$Cos\alpha(q) = -\frac{g(q)}{2\sqrt{-\left(p(q)/3\right)^3}};$$
$$\left| Cos\alpha(q) \right| < 1.$$

The final solutions for the dispersion relations are

$$\hbar\omega_1(q)=\frac{1}{3}E(q)-\frac{2}{3}\hbar\vec{v}_g\vec{q}+2\sqrt{-\frac{p(q)}{3}}Cos\frac{\alpha(q)}{3}$$

$$\hbar\omega_{2,3}(q)=\frac{1}{3}E(q)-\frac{2}{3}\hbar\vec{v}_g\vec{q}+2\sqrt{-\frac{p(q)}{3}}Cos\left(\frac{\alpha(q)}{3}\pm\frac{2\pi}{3}\right);\qquad -\left(\frac{p}{3}\right)^3>\left(\frac{g}{2}\right)^2 \tag{98}$$

$$Cos\alpha(q)=-\frac{g(q)}{2\sqrt{-\left(\frac{p(q)}{3}\right)^3}};$$

$$\hbar\omega_4(q)=E(q)$$

In the limit $q\to 0$

$$p(0)=-\left[\left(E(\vec{K})-\tilde{\mu}^{HF}\right)^2+4\left(\nu^{HF}\right)^2\right];\quad g(0)=0,\alpha(0)=\frac{3\pi}{2}$$

$$\frac{\alpha(0)}{3}=\frac{\pi}{2};\quad Cos\left(\frac{\alpha(0)}{3}\pm\frac{2\pi}{3}\right)=Cos\left(\frac{\pi}{2}\pm\frac{2\pi}{3}\right)=\mp\frac{\sqrt{3}}{2} \tag{99}$$

$$\hbar\omega_1(0)=\hbar\omega_4(0)=0;\quad \hbar\omega_{2,3}(0)=\pm\sqrt{\left(E(\vec{K})-\tilde{\mu}^{HF}\right)^2+4\left(\nu^{HF}\right)^2}$$

what coincides with formula (89). The fourth solution (86) $E(\vec{K}-\vec{P})$ equals $E(q)$ and also tends to zero when $q\to 0$.

Now the value ν and its relation with chemical potential μ will be confirmed from another side. To do it, we will start with motion equation (41) for the macroscopical large amplitude of the coherent magnetoexcitons with wave vector $\vec{K}$ neglecting the influence on it of the noncoherent quasiparticles. It has the form

$$i\hbar\frac{d}{dt}d(\vec{K})=\left(E(K)-\tilde{\mu}\right)d(K)-\nu\sqrt{N}e^{i\varphi}\left(1-2\text{v}^2\right); \tag{100}$$

$$\left(E(K)-\tilde{\mu}\right)=\left(E(K)-\bar{\mu}\right)\left(1-2\text{v}^2\right)$$

where we have put approximatively

$$\hat{\rho}(0)\cong\bar{N}_e-\bar{N}_h=0;\qquad \hat{D}(0)\approx\bar{N}_e+\bar{N}_h\approx 2\bar{N}_{ex}=2N\text{v}^2 \tag{101}$$

where v^2 is the filling factor of the LLL. Equation (100) has the form, as it was discussed by Khadzhi in this theory of coherent nonlinear light propagation in the exciton range of spectrum [30]. The time-dependent solution of equation (100) was find [30] in the form

$$\underset{\delta\to+0}{d(K,t)}=\frac{\nu\sqrt{N}e^{i\varphi}}{\left(E(K)-\bar{\mu}\right)}+Ce^{-\frac{i\left(E(K)-\tilde{\mu}\right)t}{\hbar}-\delta t} \tag{102}$$

In the limit $t\to\infty$ the damped oscillatory term vanishes and the stationary solution is established. It was determined in [4] as equal to

$$d(K)=\sqrt{N_{ex}}e^{i\varphi}=e^{i\varphi}\sqrt{N}\text{v}$$

Substituting it in equation (102) we will find in full accordance with (42)

$$\nu = \left(E(K) - \bar{\mu} \right) \mathrm{v} \tag{103}$$

It differs from expression (33) by the term $E(K)$, which is due to BEC on the state with $K \neq 0$. It is a general relation, which is true also for ν^{HF} and μ^{HF}.

In such a way we have all necessary parameters to investigate and calculate numerically the desirable dispersion relations on the base of analytical solutions (98)
obtained in the HFB approximation. The group velocity $V_g(k)$ is represented in fig.1, whereas the dispersion relations are drawn in plots 2-5, which correspond to condensate wave vectors $kl = 0$, 1;3,6 and 4,6. They are represented in three observation geometries when the wave vector $\vec{q}$ of the elementary excitation is parallel, anti parallel or perpendicular to the condensate wave vectors $\vec{k}$. There are four branches of the energy spectrum, two of which correspond to acoustical and optical plasmon branches. Other two branches belong to BEC-ed magnetoexcitons. One of them is named as quasienergy branch. Mathematically they appear due to the fact that in Bogoliubov theory of BEC side by side with the exciton annihilation operator $d(P)$ one must take also into account the complex of three operators $d^2(0)d^\dagger(-P)$. The states described by these operators have the bare energies $E_{ex}(P) = -I_l + E(P)$ and $2E_{ex}(0) - E_{ex}(-P) = -I_l - E(-P)$ correspondingly. Side by side with the branch $E(P)$ another branch $-E(-P)$ also appears, what is named as quasienergy branch.

From the physical point of view the BEC-te is nothing but an unlimited source of energy without a definite number of quanta, which permits to add or to substract to the energy quantum of any quasiparticle some energy quanta of the condensate. Just these four branches can be observed in fig.1. There are threefold degenerate branch $E(P)$ describing the two plasma branches and one energy branch of the BEC-ed magnetoexcitons. The fourth branch is a quasienergy branch and has a dispersion with the sign minus in comparison with the exciton energy branch. In the case $k = 0$ the two-dimensional magnetoexcitons form an ideal degenerate Bose gas because the interaction between the excitons without the motional dipole moments exactly equals zero. By this reason the energy branches of elementary excitations coincide with the energy spectrum of the bare noninteracting particles. The exciton-type branches of the collective elementary excitations do not contain the ionization potential I_l. It happens because in order to excite one exciton already existing in the componence of the condensate with wave vector $k = 0$ it is necessary to change the initial exciton energy $-I_l$ so as to transfer it in the state with wave vector q and final state energy $-I_l + E(q)$. The excitation energy is equal only to $E(q)$. This fact was mentioned first in [3].

When the condensate wave vector k increases so as kl equals 1;3,6 and 4,6 the attractive interaction between the magnetoexcitons appears, what makes the state of BEC-ed magnetoexcitons unstable. One can observe that in the next three figures one branch remains the same. It is not affected by the changes arised in other three branches. Only the acoustical plasmon branch is interconnected with two BEC-ed exciton branches. It results from the factorization of the fourth order determinant equation (66) and from subsequent equations (69), (71) and (77). Three remaining branches are interconnected and influence each other. When the condensate wave vector $\vec{k}$ increases it leads to the appearance of the growing attractive interaction in the system and to instabilities of the energy spectrum of the elementary excitations deduced in the frame of the HFBA. As was observed in the Introduction, the lowering of the positive energy spectrum of any branch in dependence on the wave vector means the appearing of the soft mode. It testifies that the system tends to pass in another phase.

Dispersion relations (98) for $\omega_1, \omega_2, \omega_3$ depend on the group velocity $\vec{V}_g(\vec{k})$, in the form $-\frac{2}{3}\hbar\vec{V}_g(\vec{k})\vec{q}$, what is proportional to $-(\vec{k}\bullet\vec{q})$, because $\vec{V}_g(\vec{k})$ following (90) is proportional to $\vec{k}$. Due to such structure of expressions (98) there is a supplementary negative term in one geometry, when $\vec{q}$ is parallel to $\vec{k}$. This term becomes positive in the antiparallel orientation of $\vec{q}$ and $\vec{k}$ and turns to be zero when $\vec{q}$ is perpendicular to $\vec{k}$. The negative term leads to negative values of one branch of the energy spectrum of the elementary excitations and such behavior takes place at all three values of the condensate dimensionless wave number $kl = 1; 3,6; 4,6$.

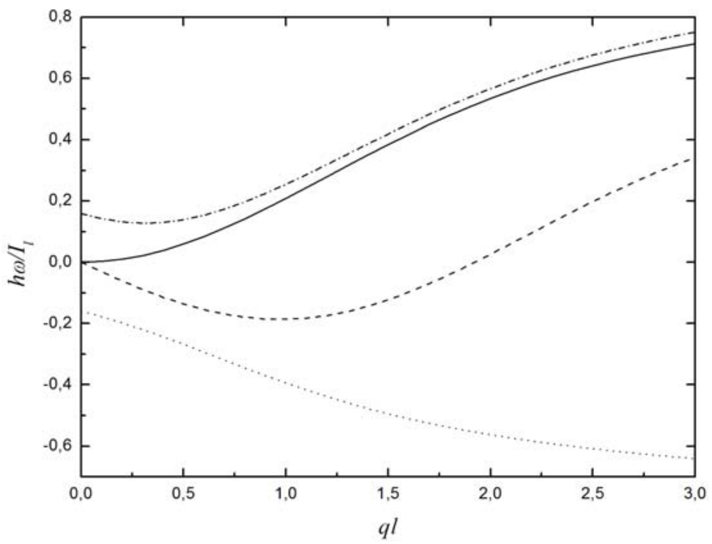
In fig.1 the group velocity $\vec{V}_g(\vec{k})$ in dependence on kl is represented. It has a maximum in the region of $kl \cong 1$.

In the next four figures the energy spectra for different values of kl as well as for different geometries of observation are drawn. One can conclude that in the HFBA all energy spectra at kl different from zero and $\mathrm{v}^2 = 0.32$ reveal the instabilities of the system due to the attractive exciton-exciton interaction.

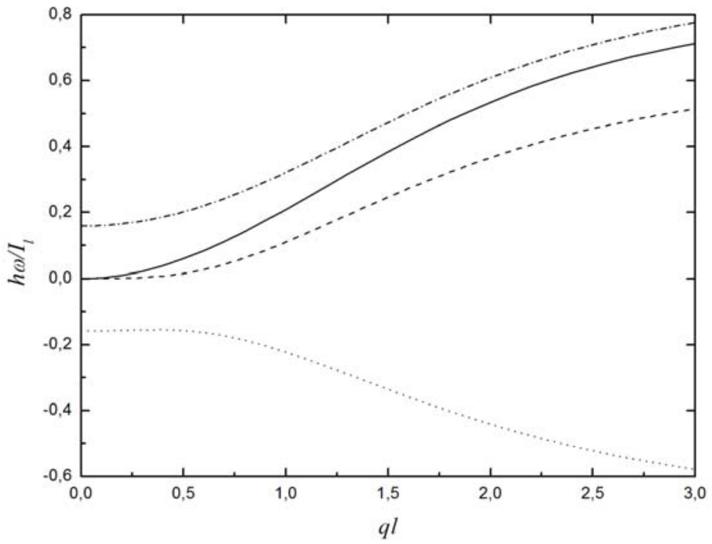
$V_g(k)/\frac{I_l l}{\hbar}$

Fig.1. The group velocity $V_g(k)$ of the magnetoexciton in units equal to $\frac{I_l l}{\hbar}$.

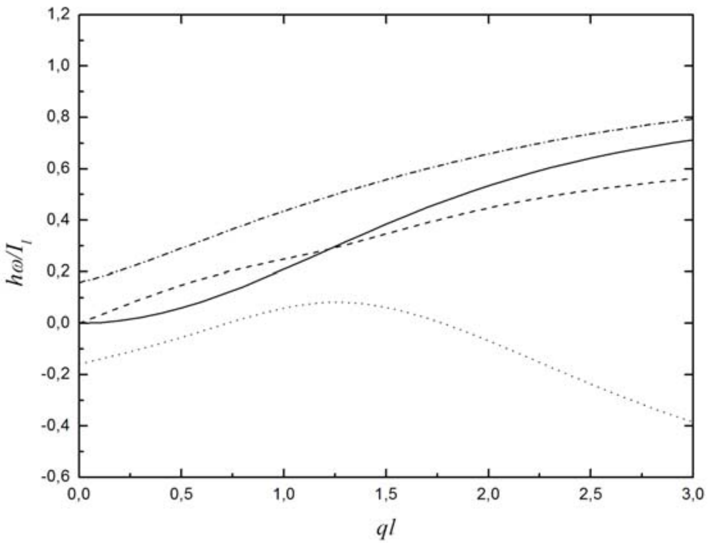
Fig.2.The energy spectrum of elementary excitations in the case when kl equals 0.
The upper branch is threefold degenerate.



a) $\vec{q}$ parallel to $\vec{k}$

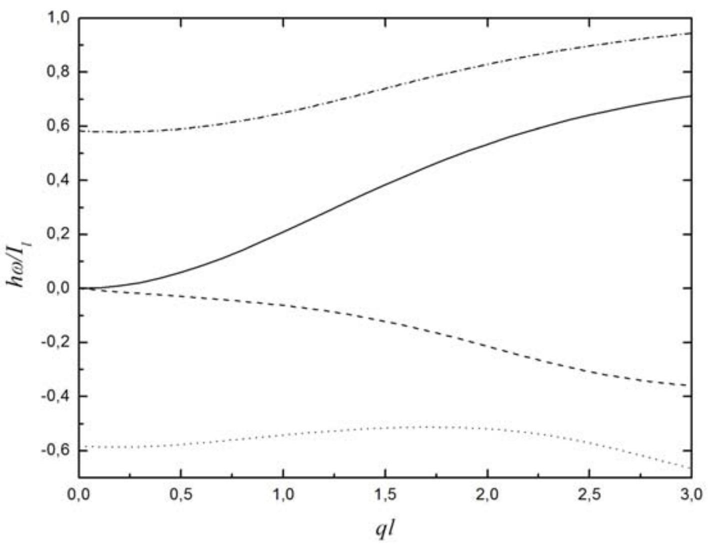


b) $\vec{q}$ perpendicular to $\vec{k}$

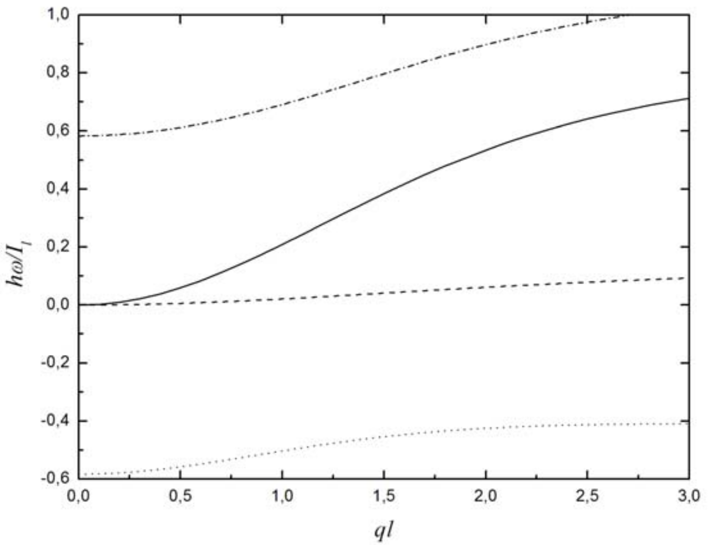


c) $\vec{q}$ antiparallel to $\vec{k}$

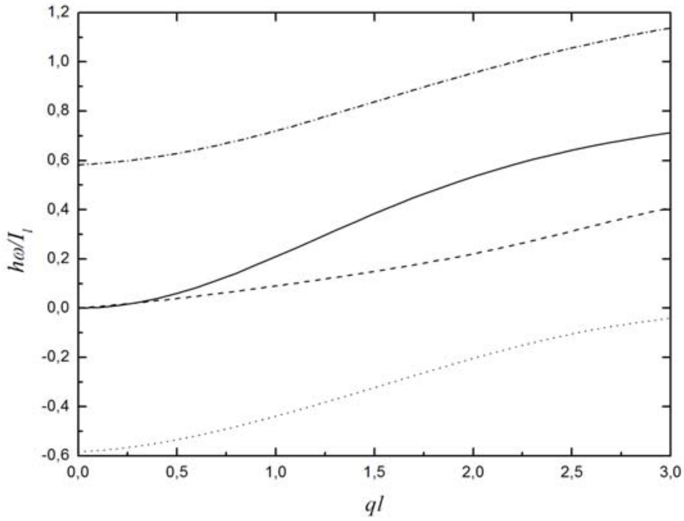
Fig.3. The energy spectrum of elementary excitations in the case when kl equals 1 for three different geometries of the observation:



a) $\vec{q}$ parallel to $\vec{k}$

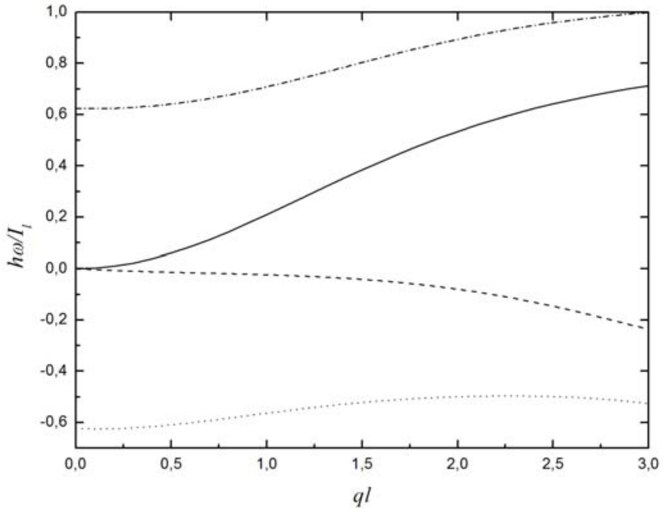


b) $\vec{q}$ perpendicular to $\vec{k}$

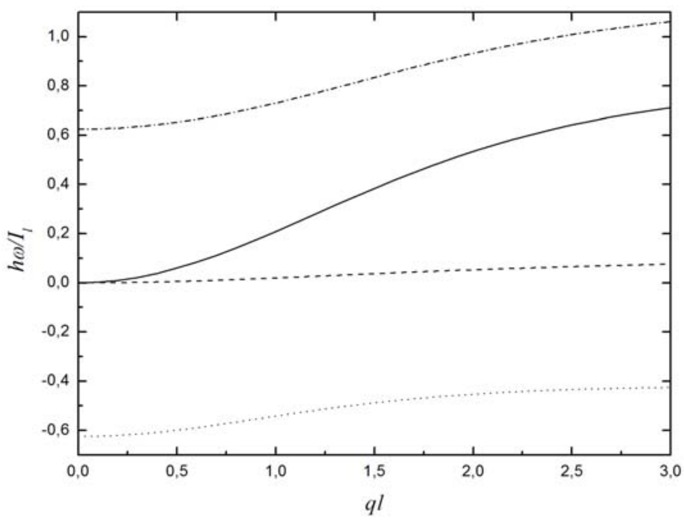


c) $\vec{q}$ antiparallel to $\vec{k}$

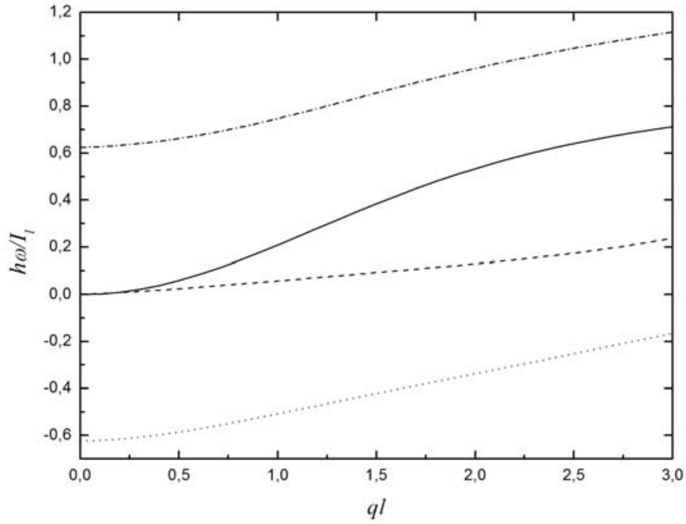
Fig.4. The energy spectrum of elementary excitations in the case when kl equals 3,6 for three different geometries of observation.



a) $\vec{q}$ parallel to $\vec{k}$



b) $\vec{q}$ perpendicular to $\vec{k}$



c) $\vec{q}$ antiparallel to $\vec{k}$

Fig.5. The energy spectrum of elementary excitations in the case when kl equals 4,6 for three different geometries of observation.

7. Self-energy parts in more complex expressions

The self-energy parts (69), (70) and (77) contain the average values of the types $\langle \hat{\rho}\hat{\rho}\rangle, \langle \hat{\rho}\hat{D}\rangle, \langle d^{\dagger}\hat{\rho}\rangle \, and \, \langle \hat{\rho}d\rangle$. They may be calculated in different approximations. Because the more important averages happened to be of the type $\langle \hat{\rho}\hat{\rho}\rangle$, we will discuss below the different approximations on the base of this example. The simpler way is to use the ground state wave function $|\psi_g(k)\rangle$ (28) of the BEC-ed magnetoexcitons and to calculate the averages in this approximation using the α_p, β_p representation instead of a_p, b_p representation because the function $|\psi_g(k)\rangle$ plays the role of vacuum state for the α_p, β_p operators.

Transforming the operators $\hat{\rho}(\vec{Q}), D(\vec{Q}), d^{\dagger}(P)$ and $d(P)$ in α_p, β_p representation and using the Wick theorem [26] we have found

$$\langle \psi_g(k) | \hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) | \psi_g(k) \rangle = 4u^2v^2 N Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)$$

$$\langle \psi_g(k) | \hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{\rho}(\vec{K}-\vec{P}-\vec{Q}) | \psi_g(k) \rangle = 4u^2v^2 N Sin^2\left(\frac{[\vec{K}\times(\vec{Q}+\vec{P})]_z l^2}{2}\right)$$

$$\langle \psi_g(k) | \hat{\rho}(\vec{Q})\hat{D}(-\vec{Q}) | \psi_g(k) \rangle = 2iu^2v^2 N Sin\left([\vec{K}\times\vec{Q}]_z l^2\right)$$

$$\langle \psi_g(k) | \hat{\rho}(\vec{Q}+\vec{P}-\vec{K})\hat{D}(\vec{K}-\vec{P}-\vec{Q}) | \psi_g(k) \rangle = 2iu^2v^2 N Sin\left([\vec{K}\times(\vec{P}+\vec{Q})]_z l^2\right)$$

$$\langle \psi_g(k) | d^{\dagger}(\vec{K}-\vec{Q})\hat{\rho}(-\vec{Q}) | \psi_g(k) \rangle \sqrt{N} = 2iuv^3 N Sin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)$$

$$\left\langle \psi_g(k) | d^{\dagger}(\vec{P}-\vec{Q})\hat{\rho}(\vec{P}-\vec{Q}-\vec{K}) | \psi_g(k) \right\rangle \sqrt{N} = 2iuv^3 NSin\left(\frac{[\vec{K}\times(\vec{P}-\vec{Q})]_z l^2}{2} \right)$$

$$\left\langle \psi_g(k) | \hat{\rho}(\vec{Q}) d(\vec{K}-\vec{Q}) | \psi_g(k) \right\rangle \sqrt{N} = -2iuv^3 NSin\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2} \right) \qquad (104)$$

$$\left\langle \psi_g(k) | \hat{\rho}(\vec{P}+\vec{Q}-\vec{K}) d(2\vec{K}-\vec{P}-\vec{Q}) | \psi_g(k) \right\rangle \sqrt{N} = -2iuv^3 NSin\left(\frac{[\vec{K}\times(\vec{P}+\vec{Q})]_z l^2}{2} \right)$$

The first two averages contain the coherence factor $Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2} \right)$, which is sign well determined and positive function at any values of wave vector $\vec{Q}$, whereas the another averages are represented by sign variable dependences and this fact will diminish significantly in some cases their contributions to the self-energy parts. The calculation of the average $\left\langle \hat{\rho}\hat{\rho} \right\rangle$ is in strong relation with the determination of the ground state energy and of correlation energy in papers [4,5].

The starting expression in these papers is

$$\frac{1}{2} W_{\vec{Q}} \left\langle 0 | \hat{\rho}(\vec{Q}) \hat{\rho}(-\vec{Q}) | 0 \right\rangle = \frac{1}{2} W_{\vec{Q}} \sum_n \left| \left(\hat{\rho}^{\dagger}(\vec{Q}) \right)_{n0} \right|^2 \quad , \qquad (105)$$

where $\left|0\right\rangle$ denotes the ground state wave function and $\left|n\right\rangle$ represents the wave function of the excited states. When the ground state wave function $\left|0\right\rangle$ was chosen in the form $\left|\psi_g(k)\right\rangle$ and the coherent excited states (46)-(56) [4] were used, they led to the expression

$$\frac{1}{2} W_{\vec{Q}} \sum_n \left| \left(\rho^{\dagger}(\vec{Q}) \right)_{n0} \right|^2 = -\int_0^{\infty} \frac{\hbar d\omega}{2\pi} \mathrm{Im}\left(\frac{1}{\varepsilon^{HF}(\vec{Q}\omega)} \right) \quad , \qquad (106)$$

which contains dielectric function of the system $\varepsilon^{HF}(\vec{Q},\omega)$ in the HF approximation. The idea suggested by Nozieres and Comte [31] and the method proposed by them is based on the affirmation that the more exact value of expression (105) can be obtained if the dielectric constant $\varepsilon^{RPA}(\vec{Q},\omega)$ in the random phase approximation (RPA) will be substituted in formula (106) instead of the value $\varepsilon^{HF}(\vec{Q},\omega)$.

This idea was applied when instead of approximation (82) in [4] the more exact expression (83) was used.

The possible ground in favour of this method is the supposition that the ground state wave function $\left|0\right\rangle$ of the BEC-ed magnetoexcitons in not exactly equal to $\left|\psi_g(k)\right\rangle$ but contains some superposition with other states, which make the variational wave function more flexible with lower energy of the ground state. As one can see from expression (2.154) [25] between new states engaged in this contribution there are the excited states with two free e-h pairs outside the condensate. On the same grounds we can expect that the more exact value of the chosen expression will be

$$\frac{1}{2} W_{\vec{Q}} \left\langle \hat{\rho}(\vec{Q}) \hat{\rho}(-\vec{Q}) \right\rangle = -\int_0^{\infty} \frac{\hbar d\omega}{2\pi} \mathrm{Im}\left(\frac{1}{\varepsilon^{RPA}(\vec{Q},\omega)} \right) \qquad (107)$$

The both expressions of the dielectric constants $\varepsilon^{HF}(\vec{Q},\omega)$ and $\varepsilon^{RPA}(\vec{Q},\omega)$ differ by their dependences on the polarizability of the system $4\pi\alpha_0^{HF}(\vec{Q},\omega)$, as follows

$$\varepsilon^{RPA}(\vec{Q},\omega) = 1 + 4\pi\alpha_0^{HF}(\vec{Q},\omega)$$
$$\frac{1}{\varepsilon^{HF}(\vec{Q},\omega)} = 1 - 4\pi\alpha_0^{HF}(\vec{Q},\omega) \tag{108}$$

In the case of BEC-ed magnetoexcitons their polarizability due to the coherent excited states in the frame of LLL approximation, without taking into account of the excited Landau levels (ELL) was deduced in [4] and has the form

$$4\pi\alpha_0^{HF}(\vec{Q},\omega) = -4u^2v^2NW_{\vec{Q}}Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\left[\frac{1}{\hbar\omega - I_{ex}(k)+i\delta} - \frac{1}{\hbar\omega + I_{ex}(k)+i\delta}\right];\delta\to 0 \tag{109}$$

It contains in the first fraction a resonant denominator, when $\hbar\omega$ equals the ionization potential $I_{ex}(k)$ of the magnetoexciton with wave vector $\vec{k}$. The singularity of expression (109) resulted in the case of correlation energy in its singular dependence of the type $\frac{1}{I_{ex}(k)}$ when $I_{ex}(k)$ tends to zero in the limit $k\to\infty$. To avoid both singularities in paper [5] instead of the infinitesimal value $\delta\to 0$ a finite value of the exciton level damping rate γ was introduced, which transforms expression (109) and its real and imaginary parts as follows

$$\begin{aligned} 4\pi\alpha_0^{HF}(\vec{Q},\omega) &= 4\pi\alpha_{0,1}^{HF}(\vec{Q},\omega) + 4i\pi\alpha_{0,2}^{HF}(\vec{Q},\omega) = \\ &= -4u^2v^2NW_{\vec{Q}}Sin^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right)\left(\Sigma(\omega,\vec{k}) + i\Gamma(\omega,\vec{k})\right), \end{aligned} \tag{110}$$

where

$$\begin{aligned} \Sigma(\omega,\vec{k}) &= \frac{\hbar\omega - I_{ex}(k)}{\left(\hbar\omega - I_{ex}(k)\right)^2+\gamma^2} - \frac{\hbar\omega + I_{ex}(k)}{\left(\hbar\omega + I_{ex}(k)\right)^2+\gamma^2}, \\ \Gamma(\omega,\vec{k}) &= \gamma\left[\frac{1}{\left(\hbar\omega + I_{ex}(k)\right)^2+\gamma^2} - \frac{1}{\left(\hbar\omega - I_{ex}(k)\right)^2+\gamma^2}\right] \end{aligned} \tag{111}$$

The needed imaginary part approximately equals

$$\mathrm{Im}\left(\frac{1}{\varepsilon^{RPA}(\vec{Q},\omega)}\right) = -4\pi\alpha_{0,2}^{HF}(\vec{Q},\omega) + 2*4\pi\alpha_{0,1}^{HF}(\vec{Q},\omega)*4\pi\alpha_{0,2}^{HF}(\vec{Q},\omega) \tag{112}$$

It leads to the desirable average value

$$\begin{aligned} W_{\vec{Q}}\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle &= -\int_0^\infty \frac{\hbar d\omega}{\pi}\mathrm{Im}\left(\frac{1}{\varepsilon^{RPA}(\vec{Q},\omega)}\right) = \\ &= 4u^2\mathrm{v}^2(NW_{\vec{Q}})Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right) - \frac{16u^4\mathrm{v}^4(W_{\vec{Q}}N)^2}{I_{ex}(k)}Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right) \end{aligned} \tag{113}$$

if the infinitesimal damping rate $\delta\to +0$ is used. The first term of (113) coincides exactly with the result (104) obtained in the HFA. The second term of (113) corresponds to correlation energy corrections, when the ground state energy is calculated. It contains the singular dependence on $I_{ex}(k)$ discussed above. Taking into account the finite exciton damping rate γ and calculating the integral

$$S(k)=\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\sum(\omega,k)\Gamma(\omega,k)=\frac{1}{2\pi}\left[\frac{\pi}{4I_{ex}(k)}-\right.$$

$$\left.-\frac{1}{2I_{ex}(k)}arctg\left(\frac{\gamma^2-I^2_{ex}(k)}{2\gamma I_{ex}(k)}\right)-\frac{\gamma}{I^2_{ex}(k)+\gamma^2}\right] \tag{114}$$

we will obtain the chosen expression without singularity

$$W_{\vec{Q}}\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle=4u^2\mathrm{v}^2\left(NW_{\vec{Q}}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)-$$

$$-\frac{16u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\left(W_{\vec{Q}}N\right)^2 Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right), \tag{115}$$

where $T(k)=4S(k)I_{ex}(k)$ has the limiting expressions

$$T(k)=1 \qquad \frac{\gamma}{I_{ex}(k)}\to 0; \qquad\qquad T(k)=\frac{2}{3\pi}\left(\frac{I_{ex}(k)}{\gamma}\right)^3 \qquad \frac{I_{ex}(k)}{\gamma}\to 0 \tag{116}$$

Now the more complete expressions for the self - energy parts will be calculated. They are listed below.

The diagonal self - energy parts $\sum_{ii}(\vec{P},\omega)$ were calculated taking also into account the terms proportional to $u^4\mathrm{v}^4W_{\vec{Q}}{}^3$ side by side with $u^2\mathrm{v}^2W_{\vec{Q}}{}^2$.

$$\sum\nolimits_{11}(\vec{P},\omega)=\hbar\omega-\tilde{\mu}+E(P)-16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}{}^2N\frac{Sin^2\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}+;$$

$$+\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}{}^3N^2\frac{Sin^2\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}$$

$$\sum\nolimits_{22}(\vec{P},\omega)=\hbar\omega+\widetilde{\mu}-E(2\vec{K}-\vec{P})-16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}^{\;2}N\frac{Sin^2\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega+\widetilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]}+$$

$$+\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}^{\;3}N^2\frac{Sin^2\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega+\widetilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta\right]};$$

$$\sum\nolimits_{33}(\vec{P},\omega)=\hbar\omega-E(\vec{K}-\vec{P})+16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\left[\left(W_{\vec{K}-\vec{P}}-W_{\vec{Q}}\right)NSin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)+\left(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}}\right)NSin^2\left(\frac{\left[\vec{K}\times(\vec{P}+\vec{Q})\right]_z l^2}{2}\right)\right]}{\left[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta\right]}-$$

$$-\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}\frac{Sin^2\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta\right]}\times$$

$$\times\left[\left(W_{\vec{K}-\vec{P}}-W_{\vec{Q}}\right)N\left(W_{\vec{Q}}N\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)+\left(W_{\vec{Q}+\vec{P}-\vec{K}}-W_{\vec{K}-\vec{P}}\right)N\left(W_{\vec{Q}+\vec{P}-\vec{K}}N\right)Sin^4\left(\frac{\left[\vec{K}\times(\vec{P}+\vec{Q})\right]_z l^2}{2}\right)\right];$$

$$\sum\nolimits_{44}(\vec{P},\omega)=\hbar\omega-E(\vec{K}-\vec{P})-16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}^{\;2}N\frac{Sin^2\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta\right]}+$$

$$+\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}^{\;3}N^2\frac{Sin^2\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta\right]}. \qquad (117)$$

The nondiagonal self - energy parts $\sum_{ij}(\vec{P},\omega)$ with $i\neq j$ do not contain the average $\left\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\right\rangle$ and in their expressions there are no terms proportional to $W_{\vec{Q}}^{\;3}$:

$$\sum\nolimits_{31}(\vec{P},\omega)=-i\nu Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\left[1+\frac{2W_{\vec{P}-\vec{K}}N}{\left[\hbar\omega-\tilde{\mu}+E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}\right]-$$

$$-\frac{4iu\mathrm{v}^3E(\vec{K})\left(W_{\vec{P}-\vec{K}}N\right)Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\left[\hbar\omega-\tilde{\mu}+E(\vec{K})-E(\vec{K}-\vec{P})+i\delta\right]}+8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{P}-\vec{K}}N\right)\times$$

$$\times\frac{Sin\left(\frac{\left[\vec{P}\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[(\vec{P}-\vec{Q})\times(\vec{P}-\vec{K})\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]}+$$

$$+8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{K}-\vec{P}}-W_{\vec{P}-\vec{K}-\vec{Q}}\right)N\frac{Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)Sin\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times(\vec{P}-\vec{Q})\right]_z l^2}{2}\right)}{\left[\hbar\omega-\tilde{\mu}+E(\vec{P}-\vec{Q})-E(-\vec{Q})+i\delta\right]};$$

$$\sum\nolimits_{32}(\vec{P},\omega)=i\nu Sin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)\left[1-\frac{2W_{\vec{K}-\vec{P}}N}{\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta}\right]-$$

$$-\frac{4iu\mathrm{v}^3E(\vec{K})W_{\vec{K}-\vec{P}}NSin\left(\frac{\left[\vec{P}\times\vec{K}\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(\vec{K})-E(\vec{K}-\vec{P})+i\delta}-8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{K}-\vec{P}}-W_{\vec{P}+\vec{Q}-\vec{K}}\right)N\times$$

$$\times\frac{NSin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{Q}\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times(\vec{P}+\vec{Q})\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta}-$$

$$-8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}W_{\vec{K}-\vec{P}}N\frac{Sin\left(\frac{\left[(2\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[(\vec{K}-\vec{Q})\times(\vec{K}-\vec{P})\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(2\vec{K}-\vec{P}-\vec{Q})-E(\vec{Q})+i\delta};$$

$$\sum\nolimits_{34}(\vec{P},\omega)=8iu^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{\left[(\vec{K}-\vec{P})\times\vec{Q}\right]_z l^2}{2}\right)\times$$
$$\times\frac{\left[\left(W_{\vec{Q}+\vec{P}-\vec{K}}\right)NSin\left(\left[\vec{K}\times(\vec{P}+\vec{Q})\right]_z l^2\right)+W_{\vec{Q}}NSin\left(\left[\vec{K}\times\vec{Q}\right]_z l^2\right)\right]}{\hbar\omega-E(\vec{Q})-E(\vec{K}-\vec{P}-\vec{Q})+i\delta}. \tag{118}$$

First of all we are interested to determine the energy spectrum of the collective elementary excitations with the wave vectors $\vec{P}$ not so far from the condensate wave vector $\vec{K}$, so that $\vec{P}=\vec{K}+\vec{q}$. There are seven more cumbersome expressions $\sum_{ij}(\vec{P},\omega)$ (117) and (118) and the remaining other simpler expressions $\sum_{ij}(\vec{P},\omega)$ (69), (70) and (77), which in dependence on $\vec{q}$ and ω have the forms $\sum_{ij}(\vec{q},\omega)$

$$\sum\nolimits_{11}(\vec{q},\omega)=\hbar\omega-\tilde{\mu}+E(\vec{K}+\vec{q})-16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}{}^2N\times$$
$$\times\frac{Sin^2\left(\frac{\left[(\vec{K}+\vec{q})\times\vec{Q}\right]_z l^2}{2}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega-\tilde{\mu}+E(\vec{K}+\vec{q}-\vec{Q})-E(-\vec{Q})+i\delta}+;$$
$$+\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}{}^3N^2\frac{Sin^2\left(\frac{\left[(\vec{K}+\vec{q})\times\vec{Q}\right]_z l^2}{2}\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega-\tilde{\mu}+E(\vec{K}+\vec{q}-\vec{Q})-E(-\vec{Q})+i\delta}$$

$$\sum\nolimits_{22}(\vec{q},\omega)=\hbar\omega+\tilde{\mu}-E(\vec{K}-\vec{q})-16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}{}^2N\times$$
$$\times\frac{Sin^2\left(\frac{\left[(\vec{K}-\vec{q})\times\vec{Q}\right]_z l^2}{2}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(\vec{K}-\vec{q}-\vec{Q})-E(\vec{Q})+i\delta}+;$$
$$+\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}{}^3N^2\frac{Sin^2\left(\frac{\left[(\vec{K}-\vec{q})\times\vec{Q}\right]_z l^2}{2}\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(\vec{K}-\vec{q}-\vec{Q})-E(\vec{Q})+i\delta}$$

$$\sum_{33}(\vec{q},\omega)=\hbar\omega-E(\vec{q})+16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)\times$$
$$\times\frac{\left[\left(W_{\vec{q}}-W_{\vec{Q}}\right)NSin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)+\left(W_{\vec{Q}+\vec{q}}-W_{\vec{q}}\right)NSin^2\left(\frac{\left[\vec{K}\times\left(\vec{Q}+\vec{q}\right)\right]_z l^2}{2}\right)\right]}{\hbar\omega-E(\vec{Q})-E(-\vec{q}-\vec{Q})+i\delta}-$$
$$-\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)\times$$
$$\times\frac{\left[\left(W_{\vec{q}}-W_{\vec{Q}}\right)N\left(W_{\vec{Q}}N\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)+\left(W_{\vec{Q}+\vec{q}}-W_{\vec{q}}\right)N\left(W_{\vec{Q}+\vec{q}}N\right)Sin^4\left(\frac{\left[\vec{K}\times\left(\vec{Q}+\vec{q}\right)\right]_z l^2}{2}\right)\right]}{\hbar\omega-E(\vec{Q})-E(-\vec{q}-\vec{Q})+i\delta};$$

$$\sum_{44}(\vec{q},\omega)=\hbar\omega-E(\vec{q})-16u^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}^{\,2}N\frac{Sin^2\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)Sin^2\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega-E(\vec{Q})-E(-\vec{q}-\vec{Q})+i\delta}+$$
$$+\frac{64u^4\mathrm{v}^4T(k)}{I_{ex}(k)}\sum_{\vec{Q}}W_{\vec{Q}}^{\,3}N^2\frac{Sin^2\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)Sin^4\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega-E(\vec{Q})-E(-\vec{q}-\vec{Q})+i\delta}. \qquad (119)$$

Three nondiagonal self-energy parts $\sum_{31}(\vec{q},\omega)$, $\sum_{32}(\vec{q},\omega)$ and $\sum_{34}(\vec{q},\omega)$ look as follows

$$\sum_{31}(\vec{q},\omega)=-i\nu Sin\left(\frac{\left[\vec{q}\times\vec{K}\right]_z l^2}{2}\right)\left[1+\frac{2W_{\vec{q}}N}{\hbar\omega-\tilde{\mu}+E(\vec{K})-E(-\vec{q})+i\delta}\right]-$$
$$-\frac{4iu\mathrm{v}^3E(\vec{K})\left(W_{\vec{q}}N\right)Sin\left(\frac{\left[\vec{q}\times\vec{K}\right]_z l^2}{2}\right)}{\hbar\omega-\tilde{\mu}+E(\vec{K})-E(-\vec{q})+i\delta}-$$
$$-8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{q}}N\right)\frac{Sin\left(\frac{\left[(\vec{K}+\vec{q})\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{q}\times\left(\vec{K}-\vec{Q}\right)\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega-\tilde{\mu}+E(\vec{K}+\vec{q}-\vec{Q})-E(-\vec{Q})+i\delta}-$$
$$-8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{q}}-W_{\vec{q}-\vec{Q}}\right)N\frac{Sin\left(\frac{\left[\left(\vec{K}+\vec{q}\right)\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times\left(\vec{q}-\vec{Q}\right)\right]_z l^2}{2}\right)}{\hbar\omega-\tilde{\mu}+E(\vec{K}+\vec{q}-\vec{Q})-E(-\vec{Q})+i\delta};$$

$$\sum\nolimits_{32}(\vec{q},\omega) = i\nu Sin\left(\frac{\left[\vec{q}\times\vec{K}\right]_z l^2}{2}\right)\left[1-\frac{2W_{\vec{q}}N}{\hbar\omega+\tilde{\mu}-E(\vec{K})-E(-\vec{q})+i\delta}\right]-$$

$$-\frac{4iu\mathrm{v}^3 E(\vec{K})\left(W_{\vec{q}}N\right)Sin\left(\frac{\left[\vec{q}\times\vec{K}\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(\vec{K})-E(-\vec{q})+i\delta}-$$

$$-8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{q}}-W_{\vec{q}+\vec{Q}}\right)N\frac{Sin\left(\frac{\left[\left(\vec{K}-\vec{q}\right)\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times\left(\vec{q}+\vec{Q}\right)\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(\vec{K}-\vec{q}-\vec{Q})-E(\vec{Q})+i\delta}-$$

$$-8iu\mathrm{v}^3\sum_{\vec{Q}}W_{\vec{Q}}\left(W_{\vec{q}}N\right)\frac{Sin\left(\frac{\left[\left(\vec{K}-\vec{q}\right)\times\vec{Q}\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{q}\times\left(\vec{K}-\vec{Q}\right)\right]_z l^2}{2}\right)Sin\left(\frac{\left[\vec{K}\times\vec{Q}\right]_z l^2}{2}\right)}{\hbar\omega+\tilde{\mu}-E(\vec{K}-\vec{q}-\vec{Q})-E(\vec{Q})+i\delta};$$

$$\sum\nolimits_{34}(\vec{q},\omega) = 8iu^2\mathrm{v}^2\sum_{\vec{Q}}W_{\vec{Q}}Sin^2\left(\frac{\left[\vec{q}\times\vec{Q}\right]_z l^2}{2}\right)\times$$

$$\times\frac{\left[\left(W_{\vec{q}+\vec{Q}}-W_{\vec{q}}\right)NSin\left(\left[\vec{K}\times\left(\vec{q}+\vec{Q}\right)\right]_z l^2\right)+W_{\vec{Q}}NSin\left(\left[\vec{K}\times\vec{Q}\right]_z l^2\right)\right]}{\left[\hbar\omega-E(\vec{Q})-E(-\vec{q}-\vec{Q})+i\delta\right]} \quad (120)$$

The remaining nine self-energy parts containing only the terms proportional to the parameter ν are

$$\Sigma_{21}(\vec{q},\omega)=0; \qquad \Sigma_{12}(\vec{q},\omega)=0;\Sigma_{43}(\vec{q},\omega)=0;$$

$$\Sigma_{41}(\vec{q},\omega)=\nu Cos\left(\frac{[\vec{q}\times\vec{k}]_z l^2}{2}\right); \qquad \Sigma_{42}(\vec{q},\omega)=-\nu Cos\left(\frac{[\vec{q}\times\vec{k}]_z l^2}{2}\right);$$

$$\Sigma_{14}(\vec{q},\omega)=2\nu Cos\left(\frac{[\vec{q}\times\vec{k}]_z l^2}{2}\right); \qquad \Sigma_{24}(\vec{q},\omega)=-2\nu Cos\left(\frac{[\vec{q}\times\vec{k}]_z l^2}{2}\right); \quad (121)$$

$$\Sigma_{13}(\vec{q},\omega)=2i\nu Sin\left(\frac{[\vec{q}\times\vec{k}]_z l^2}{2}\right); \qquad \Sigma_{23}(\vec{q},\omega)=-2i\nu Sin\left(\frac{[\vec{q}\times\vec{k}]_z l^2}{2}\right);$$

The full set of self-energy parts $\Sigma_{ij}(\vec{q},\omega)$ will be used below for the calculation of the energy spectrum of the collective excitations beyond the HFBA.

8. Energy spectrum beyond the HFBA in collinear geometry

The cumbersome dispersion equation in the form of fourth order determinant can be essentially simplified in collinear geometry when the vector product projection $[\vec{q}\times\vec{k}]_z = 0$. It takes place, when $\vec{q}$ is parallel as well is as antiparallel to condensate wave vector $\vec{k}$. At this condition two self-energy parts vanish

$$\Sigma_{13}(\vec{q},\omega) = \Sigma_{23}(\vec{q},\omega) = 0 \tag{122}$$

whereas other four self-energy parts equal to

$$\begin{aligned} &\Sigma_{14}(\vec{q},\omega) = 2\nu; \qquad &&\Sigma_{24}(\vec{q},\omega) = -2\nu; \\ &\Sigma_{41}(\vec{q},\omega) = \nu; \qquad &&\Sigma_{42}(\vec{q},\omega) = -\nu; \end{aligned} \tag{123}$$

The fourth order determinant becomes factorized in the form

$$\Sigma_{33}(\vec{q},\omega)\begin{vmatrix} \Sigma_{11}(\vec{q},\omega) & 0 & \nu \\ 0 & \Sigma_{22}(\vec{q},\omega) & -\nu \\ 2\nu & -2\nu & \Sigma_{44}(\vec{q},\omega) \end{vmatrix} = 0 \quad , \tag{124}$$

what leads to two dispersion equations. One of them is the separate equation determining the energy spectrum of an optical plasmon in the BEC-ed electron-hole system

$$\Sigma_{33}(\vec{q},\omega) = 0 \tag{125}$$

and another equation

$$\Sigma_{11}(\vec{q},\omega)\Sigma_{22}(\vec{q},\omega)\Sigma_{44}(\vec{q},\omega) - 2\nu^2(\Sigma_{11}(\vec{q},\omega) + \Sigma_{22}(\vec{q},\omega)) = 0 \tag{126}$$

determines the three interconnected branches. Two of them describe the collective elementary excitations of BEC-ed magnetoexcitons and the third branch describes the acoustical plasmon spectrum. Equation (126) is similar with equation (87) obtained in the HFBA, but their similitude is only apparent. Due to the chosen geometry the considerable simplification of the dispersion equation occurred. Below only the diagonal self-energy parts $\Sigma_{ii}(\vec{q},\omega)$ with $i = 1,2,4$ will be used, avoiding the more cumbersome components such as $\Sigma_{31}(\vec{q},\omega)$ and $\Sigma_{32}(\vec{q},\omega)$.

In spite of the condition $[\vec{q}\times\vec{k}]_z = 0$ equation (126) is not invariant under the inversion operation when $\vec{q}$ is substituted by $-\vec{q}$, because in the system there is a well selected direction determined by the BEC-ed wave vector $\vec{k}$. By this reason the elementary excitations with wave vectors $\vec{q}$ and $-\vec{q}$ have different energies.

The investigations in this direction are in progress.

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References

[1] I.V.Lerner and Yn.E.Lozovik, J.Low Temper.Phys. 38, 333 (1980)
[2] I.V.Lerner and Yn.E.Lozovik, Zh.Eksp.Teor.Fiz. 80, 1488 (1981) [Sov.Phys.-JETP 53, 763(1981)].

[3] D.Paquet, T.M.Rice and K.Ueda Phys.Rev.B, 32, 5208 (1985)
[4] S.A.Moskalenko, M.A.Liberman, D.W.Snoke and V.V.Botan Phys.Rev.B, 66, 245316 (2002)
[5] S.A.Moskalenko, M.A.Liberman, D.W.Snoke, V.V.Botan and B.Johansson Physica E, 19, 278-288 (2003)
[6] H.L.Stormer Rev.Mod.Phys. 71 №4, 875(1999)
[7] A.I.Baz`, Ya.B.Zeldovich and A.M.Perelomov Scattering, Reactions and delays in Nonrelativistic Quantum mechanics (Nauka, Moscow, 1971) (in Russian)
[8] D.Pines Elementary Excitations in Solids (Benjamin, New York, 1963)
[9] S.Das Sarma and A.Madhukar Phys.Rev.B 23, 805 (1981)
[10] S.V.Tovstonog, L.V.Kulik, I.V.Kukushkin at al. Phys.Rev.B 66 241308 (R) (2002)
[11] S.M.Girvin, A.H.MacDonald, and P.M.Platzman Phys.Rev.B 33 №4, 2481 (1986)
[12] R.B.Laughlin Phys.Rev. Lett. 50, 1395 (1981)
[13] R.P.Feynman Statistical Mechanics (Benjamin, Reading, Mass, 1972. Chap. 11)
[14] C.Kallin and B.I.Halperin Phys.Rev.B 30, 5655, 1984
[15] V.M.Apal`kov and E.I.Kashba Pis`ma Zh. Eksp.teor.Fiz. 54, 160, (1991); 55, 38, (1992)
[16] H.A.Fertig Phys.Rev.B 40, 1087, (1989)
[17] T.M.Rice, D.Paquet and K.Ueda Helv. Phys. Acta, 58, 410, (1985)
[18] Y.Kuramoto and C.Horie Solid State Commun.
[19] J.Eisenstein Abstracts of the International Conference on Spontaneous Coherence in Excitonic system. Champion, Pennsylvania, USA 25-28 May 2004, Edited by University of Pittsburgh, p.28.
[20] A.MacDonald Ibidem p.29
[21] X.M.Chen and J.J.Quinn Phys.Rev.Lett. 67, 895 (1991)
[22] X.M.Chen and J.J.Quinn Phys.Rev.B 45, 11054 (1992)
[23] N.N.Bogoliubov Collection of papers in three volumes (Naukova Dumka, Kiev, 1971) vol.2 and vol. 3 (in Russian)
[24] L.V.Keldysh and N.A.Kozlov Zv.Eksp.Teor.Fiz. 54 978 (1968) [Sov.Phys. JETP 27, 521 (1968)]
[25] S.A.Moskalenko and D.W.Snoke Bose-Einstein Condensation of Excitons and Biexcitons and Coherent Nonlinear Optics with Excitons (Cambridge University Press, Cambridge, New York, 2000)
[26] A.A.Abrikosov, L.P.Gor`kov and I.E. Dzyaloshinskii Methods of quantum field theory in statistical physics (Dover, New York, 1975).
[27] D.N.Zubarev Sov.Phys.Uspekhi Fiz.Nauk 71, 71(1960)
[28] P.W.Anderson Phys.Rev. 110, 827(1958)
[29] G.A.Korn and Th.M.Korn Mathematical Handbook for scientists and engineers Megraw-Hill Book Company N.Y. 1968
[30] P.I.Khadzhi Pis`ma Zh.Eksp.Teor.Fiz. 60, (85) (1994) [JETP Lett, 60, 89(1994)]
[31] P.Nozieres and C.Comte J.Phys. (Paris) 43 1083 (1982)