Bose-Einstein Condensation of two-dimensional magnetoexcitons. Influence of the excited Landau levels.

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Abstract

The indirect attractive interaction between the electron and holes lying on the lowest Landau levels on the surface of a two-dimensional structure in the presence of a strong perpendicular magnetic field appears due to their virtual quantum transitions to excited Landau levels as a result of the Coulomb scattering. The influence of this indirect interaction on the ground state energy and on the chemical potential of the Bose-Einstein condensed magnetoexcitons is determined. The corrections to the energy spectrum and to the wave function of the lowest magnetoexciton band due to the influence of the first three excited exciton bands were investigated.

I. Introduction

Bose-Einstein Condensation of two-dimensional (2D) magnetoexcitons was studied in the papers [1-5]. The coherent paring of electron and holes occupying only the lowest Landau levels (LLL) being situated on the surface of 2D structure in a strong perpendicular magnetic field was studied using the Keldysh-Kozlov-Kopaev method and the generalized random-phase approximation [4]. BEC of magnetoexcitons takes place on the single exciton state with wave vector $k \neq 0$, supposing that the high density of electrons in the conduction band and of holes in the valence band were created in a single quantum well (QW) structure with size quantization more greater than the Landau quantization. In the case $k \neq 0$ a new metastable dielectric liquid phase formed by BEC-ed magnetoexcitons was revealed [4,5]. It was shown that the ground state energy per exciton and the chemical potential are nonmonotonic functions on the filling factor ν^2 of the LLLs, so that the relative minimum with positive compressibility in its vicinity appears. This dielectric liquid phase is more stable than the electron-hole metallic liquid phase [4]. The polarizability of the BEC-ed magnetoexcitons was calculated using the Anderson–type wave functions of the coherent excited states. It is characterized by a coherent factor depending on k and vanishing in the case $k = 0$, as well as by a resonance frequency equal to ionization potential of a magnetoexciton with condensate wave vector k. The condensate polarizability was used to determine the correlation energy of the system and the corrections to the chemical potential beyond the Hartree-Fock-Bogoliubov approximation (HFBA). Side by side with the correlation energy conditioned by the coherent excited states the influence of the excited Landau levels on the ground

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state energy and the chemical potential was also investigated. The first estimations were made by Lerner and Lozovik [2] and a general formula without concrete calculations was proposed by Paquet, Rice and Ueda [3]. The influence of the first excited Landau levels of electrons and holes was discussed in details in the paper [5]. The indirect attraction between electron-electron (e-e), hole-hole (h-h) and electron-hole (e-h) is due to the virtual simultaneous quantum transitions of the interacting charges from LLLs to the FELLs as a result of their Coulomb scattering. The first step of the scattering and the return back to the initial states were described in the second order of the perturbation theory. Following the paper [6] due to the influence of the FELLs the coexistence of two BEC-es is possible. One of them has a wave vector $k = 0$, whereas the another one has the wave vector $k \neq 0$ in the range where the metastable dielectric liquid phase was revealed. The chemical potentials of two different BEC-es lie in the near vicinity on the energy scale at some definite values of the filling factor. If so the drops formed by dielectric liquid phase are surrounded by the degenerate Bose gas condensed on the state with $k = 0$. The task of the present paper is to continue these investigations enlarging them, so as to embrace all ELLs and not only their first level. The influence of the ELLs are many sided, but we will concentrate our attention only on two sides. One of them is related with the influence on the chemical potential of the BEC-ed magnetoexcitons. Another one concerns their influence on the wave function and on energy level of a single magnetoexciton. On this base will be possible to determine the more exact expressions of the exciton creation and annihilation operators, which in their turn play a key role in the elaboration of an a dequate theory of the BEC of magnetoexcitons. The paper is organized as follows. In the second section the simultaneous quantum transitions due to Coulomb scattering from the LLLs to the ELLs are investigated. Their influence on the chemical potential of the BEC-ed magnetoexcitons is discussed. In the third section the reciprocal influence on each other in the frame of first four magnetoexciton bands is studied with the aim to determine more exactly the wave function of the lowest magnetoexciton band. The conclusions are made in the fourth section.

II. Simultaneous quantum transitions due to the Coulomb scattering.

Now the simultaneous quantum transitions of two charged particles during their scattering under the influence of the Coulomb interaction will be considered. We have in mind the case when two particles being in the initial state on their LLLs, what means $n = m = 0$, after the Coulomb interaction happen to be on the excited states $n' = m' = n$ with the same number n and vice versa.

These processes are described by the Coulomb matrix elements

$$\begin{aligned} &F_{i-j}(p,0;q,0;p-s,n;q+s,n) \\ &F_{i-j}(p,n;q,n;p-s,0;q+s,0) \end{aligned} \qquad i,j=e,h \tag{1}$$

They determine the indirect interaction of the particles lying on the LLLs through their virtual quantum transitions to ELLs. Such indirect interaction is attractive and appears in the second order of the perturbation theory. As was demonstrated in the paper [5], where only the first excited Landau levels (FELLs) were taken into account, this indirect interaction gives rise to the shift of the magnetoexciton levels and influences on their BEC. The aim of this section is to generalize the results of the paper [5], so as to determine the influence of ELL, with the restriction $n'=m'=n$.

The general expressions for $i=j=e,h$ are

$$\begin{aligned} &F_{i-i}(p,0;q,0;p-s,n;q+s,n)=(-1)^n\sum_{\kappa}W_{s,\kappa}f(\kappa,p-q-s)\frac{(s+i\kappa)^{2n}}{2^n n!} \\ &F_{i-i}(p,n;q,n;p-s,0;q+s,0)=(-1)^n\sum_{\kappa}W_{s,\kappa}f(\kappa,p-q-s)\frac{(-s+i\kappa)^{2n}}{2^n n!} \end{aligned} \tag{2}$$

$n=0,1,2,3,4...$

These general forms were deduced explicitly for five concrete numbers $n=0,1,2,3,4$ and can be continued also for arbitrary $n\geq 5$, as we believe. The electron-hole matrix elements will be discussed below.

In the Hamiltonian (7) one can separate the term H_{Coul}^{LLL}, where only the LLLs take part and the term $\hat{H}_{Coul}^{ELL}$, which describe the simultaneous transitions $(0,0)\rightleftarrows(n,n)$ described above. The deduction of the indirect interaction between the charged particles moving in the frame of the LLLs, but undergoing the virtual transitions $(0,0)\rightleftarrows(n,n)$, is the end of this section. The importance of these quantum transitions and the study of the indirect interaction were firstly underlined and realized in the papers [2,3]. Here this question will be studied in details. Another parts of the Hamiltonian (7) will be neglected. We will separate two groups of Landau levels. One of them contains only the LLLs and the corresponding creation and annihilation operators are denoted as

$$\begin{aligned} &a_{0,p}^{\dagger}=a_p^{\dagger};a_{0,p}=a_p; \\ &b_{0,p}^{\dagger}=b_p^{\dagger};b_{0,p}=b_p; \end{aligned} \tag{3}$$

The second group contains the all excited Landau levels (ELL) with $n \geq 1$, and the corresponding operators, to be different from (3), are denoted as

$$\begin{aligned} & a^{\dagger}_{n,p} = c^{\dagger}_{n,p}; a_{n,p} = c_{n,p}; \\ & b^{\dagger}_{n,p} = d^{\dagger}_{n,p}; b_{n,p} = d_{n,p}; \\ & n \geq 1 \end{aligned} \tag{4}$$

The starting Hamiltonian under consideration is

$$\hat{H} = \hat{H}_0 + \hat{H}^{LLL}_{Coul} + \hat{H}^{ELL}_{Coul} \tag{5}$$

Here the zero order Hamiltonian H_0 is

$$\hat{H}_0 = \sum_{n=1}^{\infty}\sum_{p} n\hbar\omega_{ce} c^{\dagger}_{np} c_{np} + \sum_{n=1}^{\infty}\sum_{p} n\hbar\omega_{ch} d^{\dagger}_{np} d_{np} \tag{6}$$

The Coulomb interaction in the frame of the LLLs has the form

$$\begin{aligned} \hat{H}^{LLL}_{Coul} = & \frac{1}{2}\sum_{p,q,s} F_{e-e}(p,0;q,0;p-s,0;q+s,0) a^{\dagger}_{p} a^{\dagger}_{q} a_{q+s} a_{p-s} + \\ & + \frac{1}{2}\sum_{p,q,s} F_{h-h}(p,0;q,0;p-s,0;q+s,0) b^{\dagger}_{p} b^{\dagger}_{q} b_{q+s} b_{p-s} - \\ & - \sum_{p,q,s} F_{e-h}(p,0;q,0;p-s,0;q+s,0) a^{\dagger}_{p} b^{\dagger}_{q} b_{q+s} a_{p-s} \end{aligned} \tag{7}$$

and the virtual quantum transitions $(0,0) \rightleftarrows (n,n)$ are

$$\begin{aligned} \hat{H}^{ELL}_{Coul} = & \frac{1}{2}\sum_{p,q,s}\sum_{n} F_{e-e}(p,n;q,n;p-s,0;q+s,0) c^{\dagger}_{n,p} c^{\dagger}_{n,q} a_{q+s} a_{p-s} + \\ & + \frac{1}{2}\sum_{p,q,s}\sum_{n} F_{e-e}(p,0;q,0;p-s,n;q+s,n) a^{\dagger}_{p} a^{\dagger}_{q} c_{n,q+s} c_{n,p-s} + \\ & + \frac{1}{2}\sum_{p,q,s}\sum_{n} F_{h-h}(p,0;q,0;p-s,n;q+s,n) b^{\dagger}_{p} b^{\dagger}_{q} d_{n,q+s} d_{n,p-s} \\ & + \frac{1}{2}\sum_{p,q,s}\sum_{n} F_{h-h}(p,n;q,n;p-s,0;q+s,0) d^{\dagger}_{n,p} d^{\dagger}_{n,q} b_{q+s} b_{p-s} - \\ & - \sum_{p,q,s}\sum_{n} F_{e-h}(p,0;q,0;p-s,n;q+s,n) a^{\dagger}_{p} b^{\dagger}_{q} d_{n,q+s} c_{n,p-s} - \\ & - \sum_{p,q,s}\sum_{n} F_{e-h}(p,n;q,n;p-s,0;q+s,0) c^{\dagger}_{n,p} d^{\dagger}_{n,q} b_{q+s} a_{p-s} \end{aligned} \tag{8}$$

The expressions (7) and (8) are Hermitian conjugate, what can be verified, taking into account the properties of the Coulomb matrix elements. The condition concerning the full number of electrons and holes is not needed here, and the chemical potentials μ_e and μ_h are dropped. As earlier the energies of the spin polarized electrons and holes are accounted from the corresponding LLLs. The Hamiltonian $\hat{H}^{ELL}_{Coul}$ is considered as a first order infinitesimal as compared with $\hat{H}_0$ and

$\hat{H}_{Coul}^{LLL}$ and will be excluded from the expression (5) using the unitary transformation [9]

$$\hat{U} = e^{i\hat{S}}; \hat{S} = \hat{S}^{\dagger} \tag{9}$$

The transformed Hamiltonian

$$\begin{aligned} e^{-i\hat{S}}\hat{H}e^{i\hat{S}} &= \hat{H}_0 + \hat{H}_{Coul}^{LLL} + \hat{H}_{Coul}^{ELL} + i\left[\hat{H}_0, \hat{S}\right] + \\ &+ i\left[\hat{H}_{Coul}^{LLL}, \hat{S}\right] + i\left[\hat{H}_{Coul}^{ELL}, \hat{S}\right] - \frac{1}{2}\left[\left[\hat{H}_0, \hat{S}\right], \hat{S}\right] + \ldots \end{aligned} \tag{10}$$

contains the unknown operator $\hat{S}$, which is determined from the condition

$$i\left[\hat{H}_0, \hat{S}\right] + \hat{H}_{Coul}^{ELL} = 0 \tag{11}$$

It means, that the operator $\hat{S}$ is also a first order infinitesimal and will be determined below being proportional to the Coulomb matrix elements from $\hat{H}_{Coul}^{ELL}$.

The transformed Hamiltonian (10), following [9], must be averaged using the ground state wave function $|0\rangle_{ELL}$ of electrons and holes on the ELLs determined by the equalities

$$\begin{aligned} &c_{n,p}|0\rangle_{ELL} = d_{n,p}|0\rangle_{ELL} = 0; \\ &n \geq 1 \end{aligned} \tag{12}$$

The desirable effective Hamiltonian is [9]

$$\hat{H}_{eff} = {}_{ELL}\langle 0|e^{-i\hat{S}}\hat{H}e^{i\hat{S}}|0\rangle_{ELL} \approx \hat{H}_{Coul}^{LLL} + \frac{i}{2}{}_{ELL}\langle 0|\left[\hat{H}_{Coul}^{ELL}, \hat{S}\right]|0\rangle_{ELL} \tag{13}$$

It means that the virtual transitions $(0,0) \rightleftarrows (n,n)$ will appear only in the second order of the perturbation theory. Their operators concerning the ELLs will not take part in explicit form, but only by their averaged values. Here we took into account that

$${}_{ELL}\langle 0|\hat{H}_0|0\rangle_{ELL} = {}_{ELL}\langle 0|\left[\hat{H}_{Coul}^{LLL}, \hat{S}\right]|0\rangle_{ELL} = 0 \tag{14}$$

The condition (11) permits to determine the operator $\hat{S}$ as follows

$$\hat{S} = \hat{S}^{\dagger} = i\sum_{p,q,s}\sum_{n} \{-\frac{1}{4n\hbar\omega_{ce}} F_{e-e}(p,0;q,0;p-s,n;q+s,n) a_p^{\dagger} a_q^{\dagger} c_{n,q+s} c_{n,p-s} +$$
$$+\frac{1}{4n\hbar\omega_{ce}} F_{e-e}(p,n;q,n;p-s,0;q+s,0) c_{n,p}^{\dagger} c_{n,q}^{\dagger} a_{q+s} a_{p-s} -$$
$$-\frac{1}{4n\hbar\omega_{ch}} F_{h-h}(p,0;q,0;p-s,n;q+s,n) b_p^{\dagger} b_q^{\dagger} d_{n,q+s} d_{n,p-s} +$$
$$+\frac{1}{4n\hbar\omega_{ch}} F_{h-h}(p,n;q,n;p-s,0;q+s,0) d_{n,p}^{\dagger} d_{n,q}^{\dagger} b_{q+s} b_{p-s} +$$
$$+\frac{1}{n(\hbar\omega_{ce}+\hbar\omega_{ch})} F_{e-h}(p,0;q,0;p-s,n;q+s,n) a_p^{\dagger} b_q^{\dagger} d_{n,q+s} c_{n,p-s} -$$
$$-\frac{1}{n(\hbar\omega_{ce}+\hbar\omega_{ch})} F_{e-h}(p,n;q,n;p-s,0;q+s,0) c_{n,p}^{\dagger} d_{n,q}^{\dagger} b_{q+s} a_{p-s}\} \qquad (15)$$

Substituting the expressions (8) and (15) into (13) one will find

$$\frac{i}{2}{}_{ELL}\langle 0|\left[\hat{H}_{Coul}^{ELL}, \hat{S}\right]|0\rangle_{ELL} =$$
$$-\frac{1}{2}\sum_{p,q,s}\phi_{e-e}(p,q;p-s,q+s) a_p^{\dagger} a_q^{\dagger} a_{q+s} a_{p-s} - \frac{1}{2}\sum_{p,q,s}\phi_{h-h}(p,q;p-s,q+s) b_p^{\dagger} b_q^{\dagger} b_{q+s} b_{p-s} - \qquad (16)$$
$$-\sum_{p,q,s}\phi_{e-h}(p,q;p-s,q+s) a_p^{\dagger} b_q^{\dagger} b_{q+s} a_{p-s},$$

where

$$\phi_{i-j}(p,q;p-s;q+s) = \sum_{n=1}^{\infty}\phi_{i-j}^{n}(p,q;p-s;q+s) =$$
$$\frac{1}{\hbar\omega_{ci}+\hbar\omega_{cj}}\sum_{n=1}^{\infty}\frac{1}{n}\sum_{t} F_{i-j}(p,0;q,0;p-t,n;q+t,n) F_{i-j}(p-t,n;q+t,n;p-s,0;q+s,0) \qquad (17)$$

The indirect interaction generated by the formulas (16), (17) means the attraction between the electrons and holes. It appears in additions to the obvious Coulomb interaction described by H_{Coul}^{LLL} and represents the influence of the ELLs. In such a way we have reobtained the starting Hamiltonian (12) of the paper [5] in a more general case considering not only the first excited Landau levels (FELLs) with $n=1$, by the all of them with arbitrary $n \geq 1$. If restoring the chemical potential μ_e and μ_h, the desirable Hamiltonian is

$$\hat{H}_{eff} = -\mu_e\sum_{p} a_p^{\dagger} a_p - \mu_h\sum_{p} b_p^{\dagger} b_p + H_{Coul}^{LLL} -$$
$$-\frac{1}{2}\sum_{p,q,s}\phi_{e-e}(p,q;p-s;q+s) a_p^{\dagger} a_q^{\dagger} a_{q+s} a_{p-s} -$$
$$-\frac{1}{2}\sum_{p,q,s}\phi_{h-h}(p,q;p-s;q+s) b_p^{\dagger} b_q^{\dagger} b_{q+s} b_{p-s} - \qquad (18)$$
$$-\sum_{p,q,s}\phi_{e-h}(p,q;p-s;q+s) a_p^{\dagger} b_q^{\dagger} b_{q+s} a_{p-s}$$

As was shown in [5] the indirect $i-j$ interaction through the FELLs equals to

$$\phi_{i-j}(p,q;p-s,q+s)=\frac{1}{\hbar\omega_{ci}+\hbar\omega_{cj}}\times$$
$$\times\sum_{t}F_{i-j}(p,0;q,0;p-t,1;q+t,1)F_{i-j}(p-t,1;q+t,1;p-s,0;q+s,0) \tag{19}$$

Similarly in the case of n-th ELLs we will introduce

$$\phi^{n}_{i-j}(p,q;p-s,q+s)=\frac{1}{n(\hbar\omega_{ci}+\hbar\omega_{cj})}\times$$
$$\times\sum_{t}F_{i-j}(p,0;q,0;p-t,n;q+t,n)F_{i-j}(p-t,n;q+t,n;p-s,0;q+s,0) \tag{20}$$

Which in the case $n=1$ coincides with expression (19) and is its generalization for arbitrary values of n.

Below we will determine firstly the case $i=j=e$. Using the expressions (2) we will obtain

$$\phi^{n}_{e-e}(p,q;p-s,q+s)=\frac{1}{(2^{n}n!)^{2}}\frac{1}{2n\hbar\omega_{ce}}\times$$
$$\times\sum_{t}\sum_{\kappa}\sum_{\sigma}W_{t,\kappa}W_{s-t,\sigma}f(\kappa,p-q-t)f(\sigma,p-q-t-s)\times \tag{21}$$
$$\times(t+i\kappa)^{2n}(t-s+i\sigma)^{2n}l^{4n}$$

It is an complicate expression.

But the needed influence of the indirect interaction on the phenomenon of BEC of magnetoexcitons as well as on the metallic-type electron-hole liquid (EHL) formation is expressed by the sum of nondiagonal matrix elements of the type

$$\sum_{s}\phi^{n}_{e-e}(p,p-s;p-s,p)=\frac{1}{2n\hbar\omega_{ce}}\frac{1}{(2^{n}n!)^{2}}\times$$
$$\times\sum_{t}\sum_{k}\sum_{\kappa}\sum_{\sigma}W_{t,\kappa}W_{k,\sigma}\exp[i(\kappa k-\sigma t)l^{2}](t+i\kappa)^{2n}(-k+i\sigma)^{2n}l^{4n} \tag{22}$$

Now we introduce the two-dimensional vectors $\vec{Q}=(t,\kappa)$ and $\vec{P}=(k,\sigma)$, where $t=Qcos\varphi;\kappa=Qsin\varphi;k=Pcos\psi;\sigma=Psin\psi$ and the dimensionless variables $x=Ql$ and $y=Pl$ as well as the angle $\theta=\varphi-\psi$, what permits to express (22) in the form

$$\sum_{s}\phi^{n}_{e-e}(p,p-s;p-s,p)=\frac{I_{l}^{2}}{n\pi\hbar\omega_{ce}}\frac{1}{(2^{n}n!)^{2}}A_{2n}^{2n,2n}, \tag{23}$$

where

$$A_{2n}^{2n,2n} = \int_0^\infty dx \int_0^\infty dy x^{2n} y^{2n} e^{-\frac{(x^2+y^2)}{2}} J_{2n}(xy) \tag{24}$$

In such a way we have deduced the desirable expression (23).

$$\sum_s \phi_{e-e}^n(p, p-s; p-s, p) = \frac{I_l^2}{n\pi\hbar\omega_{ce}} \frac{1}{(n!)^2} \frac{d^{2n}}{d\alpha^{2n}} \left[\frac{1}{\sqrt{1+\alpha}} K(\kappa(\alpha)) \right]\Bigg|_{\alpha=1}, \tag{25}$$

where the modulus $\kappa(\alpha)$ of $K(\kappa)$ equals to

$$\kappa(\alpha) = \left(\frac{\alpha}{1+\alpha} \right)^{\frac{1}{2}}; \kappa(\alpha)\big|_{\alpha=1} = \frac{1}{\sqrt{2}} \tag{26}$$

and can calculate the contribution of the ELLs with $n' = m' = n$ by the final formula

$$\sum_n \sum_s \phi_{e-e}^n(p, p-s; p-s, p) = \frac{I_l^2}{\pi\hbar\omega_{ce}} \sum_{n=1}^\infty \frac{1}{n(n!)^2} \frac{d^{2n}}{d\alpha^{2n}} \left[\frac{1}{\sqrt{1+\alpha}} K\left(\left(\frac{\alpha}{1+\alpha} \right)^{\frac{1}{2}} \right) \right]\Bigg|_{\alpha=1} \tag{27}$$

In the same way we will determine the matrix elements of the electron-hole(e-h) Coulomb scattering involving the ELLs. These virtual quantum transitions from the LLLs to ELLs being taken into account in the second order of the perturbation theory will lead to supplementary attraction between the electrons and holes, increasing the binding energy of the magnetoexcitons. The virtual quantum transitions are described by the matrix elements

$$\begin{aligned} &F_{e-h}(p,0;q,0;p-s,n;q+s,n) = F_{e-h}(p,n;q,n;p-s,0;q+s,0) = \\ &= \frac{1}{2^n n!} \sum_\kappa V_{s,\kappa} \exp\left[-s^2 l^2 + i\kappa(p+q)l^2 \right] \times \\ &\times H_n\left(\frac{d}{dQ_1} \right) e^{\frac{Q_1^2}{4}} H_n\left(\frac{d}{dQ_2} \right) e^{\frac{Q_2^2}{4}} \Bigg|_{\substack{Q_1 = (s+i\kappa)l \\ Q_2 = (s-i\kappa)l}} = \frac{1}{n!} \sum_\kappa W_{s,\kappa} f(\kappa, p+q) \left[\frac{(s^2+\kappa^2)l^2}{2} \right]^n \end{aligned} \tag{28}$$

Here $H_n\left(\frac{d}{dQ} \right)$ are the Hermitian functions [3], where the variable x is substitutes by the derivative $\frac{d}{dQ}$. As earlier

$$W_{s,\kappa} = V_{s,\kappa} \exp\left[-\frac{(s^2+\kappa^2)l^2}{2} \right]; \qquad V_{s,\kappa} = \frac{2\pi e^2}{\varepsilon_0 S\sqrt{s^2+\kappa^2}}; \tag{29}$$

$$f(\kappa, p+q) = \exp[i\kappa(p+q)l^2]; \qquad I_l = \frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}};$$

The expressions (28) were verified explicitly up till the number $n=1,2,3,4$. Taking into account the matrix elements (28) one can determine in the second order of the perturbation theory the matrix element

$$\phi^{n}_{e-h}(p,q;p-s;q+s)=\frac{1}{n(\hbar\omega_{ce}+\hbar\omega_{ch})}\times \\ \times\sum_{t}F_{e-h}(p,0;q,0;p-t,n;q+t,n)F_{e-h}(p-t,n;q+t,n;p-s,0;q+s,0) \tag{30}$$

The most interest represents the sum of the nondiagonal matrix elements (30), which determines the correction to the magnetoexciton binding energy and has the form

$$\sum_{s}\phi^{n}_{e-h}(p,k_x-p;p-s;k_x-p+s)e^{-ik_y sl^2}, \tag{31}$$

where k_x,k_y are the components of the two-dimensional wave vector $\vec{k}$. They determine the states of two-dimensional magnetoexciton energy band. After cumbersome but straightforward calculations we obtained

$$\sum_{s}\phi^{n}_{e-h}(p,k_x-p;p-s;k_x-p+s)e^{-ik_y sl^2}= \\ =\frac{2I_l^2(A_n(kl))^2}{2^{2n}(n!)^2 n\pi(\hbar\omega_{ce}+\hbar\omega_{ch})}, \tag{32}$$

where [8]

$$A_n(kl)=\int_0^{\infty}x^{2n}e^{-\frac{x^2}{2}}J_0(xkl)dx=2^{\left(n-\frac{1}{2}\right)}\Gamma\left(n+\frac{1}{2}\right)e^{-\frac{k^2l^2}{2}}{}_1F_1\left(\frac{1}{2}-n;1;\frac{k^2l^2}{2}\right) \tag{33}$$

In the case $n=1$ this expression coincides exactly with the formula (33) of the paper [5].
The summary contribution of different ELLs is expressed by the sum

$$\sum_{n=1}^{\infty}\sum_{s}\phi^{n}_{e-h}(p,k_x-p;p-s;k_x-p+s)=\frac{2I_l^2}{\pi(\hbar\omega_{ce}+\hbar\omega_{ch})}\sum_{n=1}^{\infty}\frac{\left(A_n(kl)\right)^2}{2^{2n}(n!)^2 n} \tag{34}$$

The expressions (27) and (34) will be used to determine the influence of the ELLs on the position on the energy scale of the magnetoexciton level as well as of the stability of their Bose-Einstein condensation on the single-particle state with wave vector $\vec{k}$.

III. Wave functions and energy spectrum of four magnetoexciton bands.

For the beginning we will determine the magnetoexciton creation operators $d^{\dagger}_{n,m,\vec{k}}$ characterized by the number n of the electron Landau level and by the number m of the hole Landau level, as well as by the two – dimensional wave vector $\vec{k}$ with two components k_x and k_y. They are

$$d^{\dagger}_{n,m,\vec{k}}=\frac{1}{\sqrt{N}}\sum_{t}e^{-ik_y tl^2}a^{\dagger}_{n,\frac{k_x}{2}+t}b^{\dagger}_{m,\frac{k_x}{2}-t} \tag{35}$$

The state with the pair of numbers (n,m) equals to (0,0) will for simplicity be denoted as the state 1, the pair of numbers (1,1) gives rise to the magnetoexciton state 2, the pairs (1,0) and (0,1)

will be mentioned as the states 3 and 4. The exciton wave function $\psi_i(k)$ are obtained acting by these operators on the vacuum wave function $|0\rangle$ determined as

$$a_{n,p}|0\rangle = 0; \qquad b_{m,p}|0\rangle = 0 \tag{36}$$

These functions are

$$\psi_{i\bar{k}} = d^{\dagger}{}_{n,m,k}|0\rangle; \qquad i \to (n,m) \tag{37}$$

They obey to the orthogonality and normalization conditions

$$\langle \psi_{i\bar{k}} | \psi_{j\bar{k}'} \rangle = \delta_{i,j}\delta_{k\bar{r}\bar{k},\bar{k}'} \tag{38}$$

The Hamiltonian consists from two parts $H_0(6)$ and $H_{coul}(7)$, where the chemical potentials μ_e and μ_h are dropped, because we will study the energy spectrum of a single magnetoexciton. The quantum statistical properties of the magnetoexcitons here will be not discussed and the condition of a given number of magnetoexcitons it is not necessary.

The matrix elements of the Hamiltonian on the wave functions (37) are denoted as

$$H_{ij}(k) = \langle \psi_{ik} | \hat{H} | \psi_{jk} \rangle = \langle \psi_{ik} | \hat{H}_0 | \psi_{jk} \rangle + \langle \psi_{ik} | \hat{H}_{coul} | \psi_{jk} \rangle = \delta_{i,j} \langle \psi_{ik} | \hat{H}_0 | \psi_{ik} \rangle + V_{i,j}(k) \tag{39}$$

Firstly the magnetoexciton energy bands are determined in zero order of the perturbation theory, when only the diagonal matrix elements and two off-diagonal matrix elements $V_{12}(k)$ and $V_{21}(k)$ are taken into account. All other off-diagonal matrix elements, which are smaller than the values V_{12} and V_{21} will be introduced in the higher orders of the perturbation theory.

The four zero order magnetoexciton bands accounted from the LLLs are

$$\begin{aligned} E^1{}_{ex}(k) &= E^{(0,0)}{}_{ex}(k) = H_{1,1}(k) = V_{1,1}(k) = -I^{(0,0)}{}_{ex}(k) \\ E^2{}_{ex}(k) &= E^{(1,1)}{}_{ex}(k) = H_{2,2}(k) = \hbar\omega_{ce} + \hbar\omega_{ch} - I^{(1,1)}{}_{ex}(k) \\ E^3{}_{ex}(k) &= E^{(1,0)}{}_{ex}(k) = H_{3,3}(k) = \hbar\omega_{ce} - I^{(1,0)}{}_{ex}(k) \\ E^4{}_{ex}(k) &= E^{(0,1)}{}_{ex}(k) = H_{4,4}(k) = \hbar\omega_{ch} - I^{(0,1)}{}_{ex}(k) \end{aligned} \tag{40}$$

The ionization potentials of four bands were determined as

$$\begin{aligned} I^{(n,m)}{}_{ex}(k) &= \sum_s F_{e-h}(p,n;k_x - p,m;p-s,n;k_x - p + s,m)e^{ik_y sl^2} = \\ &= \frac{e^2}{\varepsilon_0 l}\int_0^{\infty} dx e^{-\frac{x^2}{2}}(1-\frac{x^2}{2})^{n+m} J_0(klx); \end{aligned} \tag{41}$$

where $n,m = 0,1$

The ionization potentials are

$$I^{(0,0)}{}_{ex}(k) = I_l e^{-\frac{k^2 l^2}{2}}\,{}_1F_1(\frac{1}{2},1;\frac{k^2 l^2}{2}) = I_l e^{-\frac{k^2 l^2}{4}} I_0(\frac{k^2 l^2}{4}); I^{(0,0)}{}_{ex}(0) = I_l,$$

$$I^{(1,1)}{}_{ex}(k) = I_l e^{-\frac{k^2 l^2}{2}}\left[{}_1F_1(\frac{1}{2},1;\frac{k^2 l^2}{2}) + \frac{3}{4}\,{}_1F_1(-\frac{3}{2},1;\frac{k^2 l^2}{2}) - {}_1F_1(-\frac{1}{2},1;\frac{k^2 l^2}{2})\right];$$

$$I^{(1,1)}{}_{ex}(0) = \frac{3}{4} I_l, \qquad (42)$$

$$I^{(1,0)}{}_{ex}(k) = I^{(0,1)}{}_{ex}(k) = I_l e^{-\frac{k^2 l^2}{2}}\left[{}_1F_1(\frac{1}{2},1;\frac{k^2 l^2}{2}) - \frac{1}{2}\,{}_1F_1(-\frac{1}{2},1;\frac{k^2 l^2}{2})\right];$$

$$I^{(1,0)}{}_{ex}(0) = I^{(0,1)}{}_{ex}(0) = \frac{1}{2} I_l$$

Here ${}_1F_1(\alpha,\gamma,z)$ are the degenerate hypergeometric functions [8] with the property

$${}_1F_1(\alpha,\gamma;-z) = e^{-z}\,{}_1F_1(\gamma-\alpha;\gamma,z) \qquad (43)$$

and $I_0(z)$ is the modified Bessel function with the limits [8]

$$I_0(z) \underset{z\to 0}{=} \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}; \qquad I_0(z) \underset{z\to\infty}{=} \frac{e^z}{\sqrt{2\pi z}} \qquad (44)$$

One can observe that the ionization potential of four types magnetoexciton bands equal to I_l, $3/4\, I_l$ and $1/2\, I_l$ correspondingly. They are determined only by the diagonal Coulomb matrix elements. There are 12 off-diagonal matrix elements $V_{i,j}$ with $i \neq j = 1,2,3,4$, which can be represented as follows [7]

$$V_{12}(k)=V_{21}(k)=-\sum_{\vec{Q}}V_{\vec{Q}}\frac{Q^2l^2}{2}\exp\left[-\frac{Q^2l^2}{2}+i\left[\vec{k}\times\vec{Q}\right]_z l^2\right]=$$

$$=-I_l\sqrt{\frac{2}{\pi}}\int_0^\infty dx\frac{x^2}{2}e^{-\frac{x^2}{2}}J_0(klx)=-I_l\frac{1}{2}e^{-\frac{k^2l^2}{2}}{}_1F_1(-\frac{1}{2},1;\frac{k^2l^2}{2});$$

$$V_{13}(k)=V^*_{31}(k)=-V_{41}(k)=-V^*_{14}(k)=$$

$$=-\sum_{\vec{Q}}V_{\vec{Q}}\frac{(s+i\kappa)l}{\sqrt{2}}\exp\left[-\frac{Q^2l^2}{2}+i\left[\vec{k}\times\vec{Q}\right]_z l^2\right]=$$

$$=I_l e^{i\psi}\int_0^\infty dxe^{-\frac{x^2}{2}}xJ_1(klx)=\frac{e^{i\psi}}{2\sqrt{2}}(kl)I_l e^{-\frac{k^2l^2}{2}}{}_1F_1(\frac{1}{2},2;\frac{k^2l^2}{2});$$

$$V_{32}(k)=V^*_{23}(k)=-V_{24}(k)=-V^*_{42}(k)=$$

$$=\sum_{\vec{Q}}V_{\vec{Q}}\frac{(i\kappa-s)l}{\sqrt{2}}\left(1-\frac{Q^2l^2}{2}\right)\exp\left[-\frac{Q^2l^2}{2}+i\left[\vec{k}\times\vec{Q}\right]_z l^2\right]=$$

$$=-I_l e^{-i\psi}\frac{1}{\sqrt{\pi}}\int_0^\infty dxe^{-\frac{x^2}{2}}x\left(1-\frac{x^2}{2}\right)J_1(klx)=$$

$$=-\frac{e^{-i\psi}}{2\sqrt{2}}(kl)I_l e^{-\frac{k^2l^2}{2}}\left[{}_1F_1(\frac{1}{2},2;\frac{k^2l^2}{2})-\frac{3}{2}{}_1F_1(-\frac{1}{2},2;\frac{k^2l^2}{2})\right];$$

$$V_{34}(k)=V^*_{43}(k)=\sum_{\vec{Q}}V_{\vec{Q}}\frac{(i\kappa-s)^2l^2}{\sqrt{2}}\exp\left[-\frac{Q^2l^2}{2}+i\left[\vec{k}\times\vec{Q}\right]_z l^2\right]=$$

$$=e^{-2i\psi}I_l\frac{1}{2\pi}\int_0^\infty dxe^{-\frac{x^2}{2}}x^2J_2(klx)=\frac{3}{16}e^{-2i\psi}(kl)^2I_l e^{-\frac{k^2l^2}{2}}{}_1F_1(\frac{1}{2},3;\frac{k^2l^2}{2})$$

(45)

Here the two – dimensional vector $\vec{Q}$ has the components s and κ $\vec{Q}=(s,\kappa)$ and ψ is the angle, which determines the components of the wave vector $\vec{\kappa}=(\kappa_x=\kappa\cos\psi,\kappa_y=\kappa\sin\psi)$. But the final results will not depend on the angle ψ and it can be dropped.

The more exact expression of the magnetoexciton wave function is the linear combination

$$\psi_{\nu,k}=\sum_{i=1}^{4}a_{i\nu}\psi_{ik} \tag{46}$$

The new functions $\psi_{\nu,k}$ are denoted by Greek symbol $\nu=\alpha,\beta,\gamma,\delta$, whereas the initial zero order functions ψ_{ik} by Latin letter i=1,2,3,4. They obey to the Schrödinger equation

$$\hat{H}\psi_{\nu k}=E_\nu(k)\psi_{\nu k} \tag{47}$$

The final expression for the magnetoexciton energy bands $E_\nu''(k)$ in dependence of the dimensionless wave vector kl was obtained in second order of the perturbation theory, when the quantum number ν equals to α, β:

$$E_\nu''(k) = \left[\frac{|V_{13}(k)|^2}{E_\nu^0(k) - H_{33}(k)} + \frac{|V_{14}(k)|^2}{E_\nu^0(k) - H_{44}(k)} \right] \left| a_{1\nu}^0 \right|^2 +$$

$$+ \left[\frac{|V_{23}(k)|^2}{E_\nu^0(k) - H_{33}(k)} + \frac{|V_{24}(k)|^2}{E_\nu^0(k) - H_{44}(k)} \right] \left| a_{2\nu}^0 \right|^2 + \qquad , \quad \nu = \alpha, \beta \qquad (48)$$

$$+ a_{1\nu}^{0*} a_{2\nu}^0 \left[\frac{V_{13}(k) V_{32}(k)}{E_\nu^0(k) - H_{33}(k)} + \frac{V_{14}(k) V_{42}(k)}{E_\nu^0(k) - H_{44}(k)} \right] +$$

$$+ a_{2\nu}^{0*} a_{1\nu}^0 \left[\frac{V_{23}(k) V_{31}(k)}{E_\nu^0(k) - H_{33}(k)} + \frac{V_{24}(k) V_{41}(k)}{E_\nu^0(k) - H_{44}(k)} \right]$$

Conclusions.

The states of excitons on the surface of an ideal symmetric two-dimensional (2D) layer in a strong perpendicular magnetic fields are studied taking into account explicitly five levels of Landau quantization for electrons $n_e = 0,1,2,3,4$ and five levels for holes $n_h = 0,1,2,3,4$. One their examples the recurrent formulas for the Coulomb matrix elements for arbitrary n_e and n_h were deduced. The 2D magnetoexcitons are formed by the electron-hole (e-h) pairs, in which the electron occupies the Landau levels with number n_e, whereas the hole occupies the Landau level with number n_h. Their binding takes place due to Coulomb interaction, which is supposed to be smaller than the distances $\hbar\omega_{ce}$ and $\hbar\omega_{ch}$ between the Landau levels. The exciton states in our description are characterized in Landau gauge by the pair (n_e, n_h) of quantum numbers, which describe the Landau quantization in one in-plane direction and by two uni-dimensional wave vectors p, q describing the motion of electron and hole in another perpendicular in-plane direction. Instead of them the 2D in-plane wave vector $\vec{k} = (k_x, k_y)$ is introduced, which characterizes the motion of the exciton as a whole and its continuous energy spectrum of the e-h relative motion. Two main topics concerning the 2D magnetoexcitons are discussed. One of them is the influence of the excited Landau levels on the collective properties of the high density magnetoexcitons. Starting with the explicit expressions of their wave functions the matrix elements of the Coulomb interactions in the e-h system were determined. The recurrent formulas were obtained, which permit to determine in the second order of the perturbation theory the indirect interaction between the electrons and holes situated on the lowest Landau levels. This indirect interaction changes the magnetoexciton ionization potential and the chemical potential of the Bose-Einstein condensed two-dimensional magnetoexcitons.

The second question discussed in the paper is the energy spectrum of four lowest exciton bands. Their wave functions were determined in the first order of the perturbation theory, whereas their energy spectrum in the second order of the perturbation theory. As the perturbations some nondiagonal matrix elements of the Coulomb interaction are considered. They are much less than the diagonal matrix elements.

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