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Constrained Bézier curves¹

In this paper we present two new classes of n th degree Bézier curves while its control points \mathbf{b}_{n-1} or \mathbf{b}_1 are always belong to a curve. Degenerate Bézier curves may be considered as a particular case of presented class.

A Bézier curve is a parametric curve frequently used in computer graphics and related fields to model smooth curves that can be scaled indefinitely.

As usual it is written as follows

$$\mathbf{b}^n(t) = \sum_{i=0}^n \mathbf{b}_i B_i^n(t),$$

where $B_i^n(t)$ are Bernstein polynomials, defined explicitly by

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}.$$

Let us consider that a curve () has a control point $\mathbf{b}_{n-1} \in \mathbf{b}^n(t) \forall t \in [0, 1]$, then it is possible to write

$$\sum_{i=0}^n \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} = \mathbf{b}_{n-1},$$

$$\begin{aligned} \sum_{i=0}^{n-2} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} + \mathbf{b}_{n-1} \binom{n}{n-1} t^{n-1} (1-t) + \\ + \mathbf{b}_n \binom{n}{n} t^n = \mathbf{b}_{n-1}, \end{aligned}$$

and it follows that

$$\mathbf{b}_{n-1} = \frac{1}{1 - nt^{n-1}(1-t)} \left(\sum_{i=0}^n \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} + \mathbf{b}_n t^n \right).$$

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Further we consider that a Bézier curve has a control point $\mathbf{b}_1 \in \mathbf{b}^n(t) \forall t \in [0, 1]$, then

$$\sum_{i=0}^n \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} = \mathbf{b}_1,$$

$$b_0 \binom{n}{0} t^0 (1-t)^n + b_1 \binom{n}{1} t (1-t)^{n-1} +$$

$$\sum_{i=2}^n \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} = \mathbf{b}_1,$$

and finally we obtain

$$\mathbf{b}_1 = \frac{1}{1 - nt^{n-1}(1-t)} \left(\sum_{i=2}^n \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} + \mathbf{b}_0 (1-t)^n \right).$$

The following figures show the first class of a cubic constrained Bézier curve.

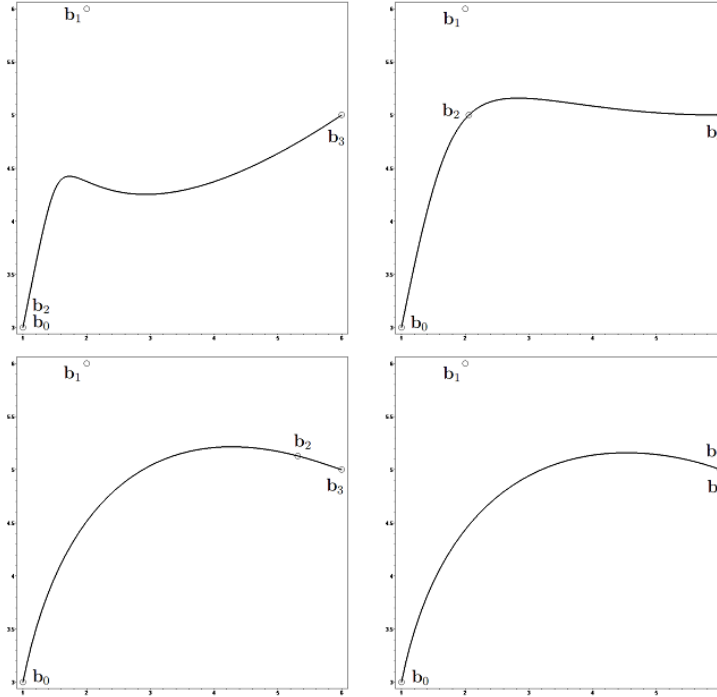


Fig. 1. Constrained Bézier curves at $t = 0, 0.4, 0.8, 1$. The last figure shows the degenerate Bézier curve.

REFERENCES

1. Gerald Farin. *Curves and Surfaces for CAGD. A Practical Guide*. Academic Press, 2002. – 499 P.